Optimal income taxation in the presence of networks of altruism*

Anasuya Raj†

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Abstract

Following the tradition of Mirrlees (1971), static optimal income taxation problems are typically concerned with public redistribution by the State between independent individuals or couples. This paper is the first to depart from this independence assumption and to account for the interaction between public redistribution and private transfers occurring between individuals within their familial and social networks. Here, individuals value the private utilities of their group's members and differ in their productivity levels. As a result, they have two dimensions of choice: their productive effort and the transfers they make to other members of their network. I characterize theoretically the optimal linear and non-linear income tax schemes in this context and provide sufficient statistics formulas for both, which highlight how tax design is affected by the existence of altruistic private transfers. I show that public and private redistribution are substitutes but at the optimum, private transfers are only partially crowded-out: the government may rely on these transfers to achieve its equity objective. Depending on the structure of groups, taking into account private redistribution may induce large efficiency gains – a finding I illustrate using numerical simulations. These novel insights can then be applied to a variety of settings, such as those of family taxation and of taxation in developing countries, for which I document the structure and nature of informal transfers using the World Bank's Living Standard Measurement Surveys.

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JEL classification: D31, H21, H3, D64, D85.

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†CREST, École Polytechnique, 5 avenue Henry le Chatelier, 91120 Palaiseau, France. E-mail: anasuya.raj@polytechnique.edu

1 Introduction

Following the tradition of Mirrlees (1971), static optimal income taxation problems are typically concerned with redistribution by the State between independent individuals. However, as poet John Donne wrote, "no man is an island entire of itself". In practice, individuals have links with one another — with their family, friends, neighbors or with members of a community —, and are thus part of networks through which information, money or help circulate.[1] As a matter of fact, governments often take into account such ties in the design of income tax schedules by, for example, taxing individuals at a household level.[2] Yet little has been done in the economics literature to understand the interaction between familial and social networks and optimal income taxation.

This paper is the first to analyze the design of optimal income taxation in the presence of networks of altruism. The existence of altruism between individuals is well-grounded in the academic literature, be it in economics, sociology or anthropology. Starting with Becker (1974; 1981), it has given rise to a large literature in economics – both theoretical and empirical.[3] Altruism can take different forms in developed and developing countries, and in particular, may expand beyond close family circles. In developing countries, these kinship networks may encompass individuals from one's extended family, village, or even caste or community (see Cox and Fafchamps (2007), Munshi (*forthcoming*)). In developed countries, even though altruism is generally thought of as existing primarily between parents and children, evidence suggests it is also present in extended families – especially between grand-parents and grand-children. The recent rise of individual or even nation-based crowd funding might constitute further evidence for altruism being present beyond family circles. The design of income taxation, in turn, should be concerned with altruism as it has important theoretical implications. Indeed, altruism is one of the principal underlying reasons for private transfers between individuals – inter-households, or intra-household.[4] Moreover, altruistically motivated transfers are redistributive: they flow from richer to poorer households. These private transfers thus represent a form of informal redistribution within networks and raise the question of their interaction with public redistribution.

A straightforward postulate might be that redistribution can be achieved in the economy without State intervention by simply letting monetary transfers flow from richer to poorer households within networks. However, individuals who receive transfers from their networks do not necessarily coincide with the agents to whom the State would want to redistribute. Indeed, an individual might be poorer relative to the other members of their network, but much richer than members of other networks.

The mirror postulate would be that the State should completely crowd out private transfers and design its income tax without accounting for these. The essence of income taxation problems, however, is that endogenous effort decisions are unobservable by the State – which creates an asymmetry

[1] These links were analyzed and theorized upon as early as the Greek and Roman eras, by Cicero for example, and more recently by philosophers such as Emmanuel Mounier, and more remarkably Georg Simmel. In economics, social networks are increasingly studied, see Jackson, (2010; 2019).

[2] For example, the "foyer fiscal" in France, the "tax unit" in the US, ...

[3] See for example, Arrow (1981), Cox (1987), Cox, Hansen and Jimenez (2004) Cox, Galasso and Jimenez (2006), Farhi and Werning (2013), Alger and Weibull (2010), Bourlès, Bramoullé and Perez-Richet (2017), Bourlès, Bramoullé and Perez-Richet (2018) and the handbook chapter by Mercier Ythier (2006).

[4] See Rapoport and Docquier (2006) for a survey on the different motives for informal transfers.

of information between the agents and the State. Since the State cannot observe exogenous productivity levels (and endogenous productive effort), it has to redistribute as a function of income, thus distorting productive effort decisions and generating an efficiency loss. This is the equity-efficiency trade-off. Higher tax rates enable more redistribution, but come at a cost: marginal tax rates distort agents' incentives to exert effort, which then reduces the size of the total income that the State can split between agents. Altruistic transfers, on the other hand, either do not distort productive effort decisions, or encourage more productive individuals to work more (and less productive ones to work less). Hence, private altruistic redistribution comes with no total (productive) efficiency loss, and even potentially an efficiency gain – thus impacting the classic equity-efficiency trade-off.

The presence of private altruistically motivated transfers in the society therefore raises a number of questions, notably: how do these two modes of redistribution – by the State and within networks – co-exist? And how ought this to affect income tax design by the State? I first develop a baseline framework, that gives some key insights into how altruistic private transfers affect the classic equity-efficiency trade-off of the Mirrlees model. I subsequently expand the model by relaxing some of the initial assumptions and consider related applications. Throughout, I illustrate my results with simulations.

In the baseline framework, I account for altruistic links between individuals in an otherwise standard Mirrleesian framework, where (i) individuals are endowed with exogenous productivity levels that are unobservable to the State, and (ii) the State seeks to maximize a social welfare function that reflects its redistributive preferences. Individuals are altruistic and thus value the private utility of their peers. They then play a non-cooperative game in which they choose their productive effort as well as the transfers they make. I assume that the network of altruism is composed of different groups – for example, extended families, or communities – in order to account for heterogeneity in productivity levels within and across groups. I then derive the structure of incomes and transfers absent government intervention and find that redistribution may take place at the group level. Depending on how representative of society groups are, this may induce more or less redistribution society-wide. This is the context in which the government introduces an income tax in order to raise revenue and redistribute between individuals. I find that the formal system relies on the informal one, and at the optimum both positive tax rates and positive transfers co-exist. I characterize theoretically the optimal linear and non-linear income tax schemes. The sufficient statistics formula for marginal tax rates still depends on the distribution of skills in the society, the social planner's redistributive preferences and the elasticity of earnings to the net of tax rate, and is appropriately modified to include the effect of taxes on private transfers.

More precisely, I build on the setting of Bourlès et al. (2017), where individuals are all part of a network, in which they value the private utility of the agents they are linked to. In my setting, the structure of the network of altruism is such that individuals belong to different components of a network, and each component is complete as in Arrow (1981).[5] Within each component there are different distributions of individual skills. We refer to these components as groups. Unlike Bourlès et al. (2017) and Arrow (1981), however, earnings are endogenous. Thus, individuals should not only choose the

[5]The adjacency matrix of the network of altruism is thus bloc-diagonal.

transfers they make, but also their productive effort. The group structure enables me to consider two layers of heterogeneity. First, heterogeneity between agents: agents differ in their productivity levels and in the groups they belong to. Both determine an agent's final consumption. Second, heterogeneity between groups: groups may be composed of higher or lower productivity agents on average, so that an individual considered as low skilled in one group may be the one with the highest skill in another group. Finally, I model the individuals' choices as a simultaneous non-cooperative game, where individuals choose their productive effort and transfers taking the choices of the other individuals as given.

I characterize the economy when there is no government intervention. I prove that there exists a Nash equilibrium to the game described above and that it uniquely determines allocations.[6] This induces a new network, one of transfers. Quite intuitively, transfers do not take place between individuals of different groups. Moreover, I find that transfers only flow from higher skilled to lower skilled individuals, and do so only if altruism is high enough or if skill levels are different enough. Private transfers are thus redistributive: within groups, they reduce the consumption gap between higher and lower skilled individuals. All individuals of the same productivity level and in the same group behave identically: they provide the same productive effort and make or receive the same sum of transfers.

Secondly, I characterize the optimal income tax in such a setting assuming that the government can observe neither productivity levels nor group belonging. The setting is a standard Mirrleesian one, in which, however, individuals are altruistically linked.

Under standard assumptions on the shape of utility functions, I first show that a linear tax on income partially crowds out private transfers. Indeed, the effect of a linear income tax on transfers can be decomposed into two. Through lump-sum redistribution it increases all individuals' available incomes and through the linear income tax rate, it decreases the available incomes for higher skilled individuals more than it does for lower skilled ones. Both effects decrease altruistic transfers. The linear income tax, however, distorts productive effort decisions, whereas private altruistic transfers do not. The optimal income tax formula thus still depends on the redistributive tastes of the Social Planner and on the elasticity of earnings to the net-of-tax rate but should be adjusted downward to take into account the derivative of transfers with respect to the tax rate. In particular, two interesting cases arise either when transfers flow from the very rich to the very poor, or when they only take place in a wider middle class. In the latter case, the tax rate might hardly adjust to transfers: the government faces two forces that go in opposite directions. Lowering the tax rate enables less productive effort distortion and more redistribution within the middle income group, but it does not enable enough redistribution from the very rich to the very (isolated) poor. These results stand in contrast with Chetty and Saez (2010). In their paper, the objective functions of the State and of a central planner in the informal economy (the private insurer) coincide. In my setting, even though there is no moral hazard between individuals, the equilibrium of the non-cooperative game is not Pareto efficient, hence does not coincide with the optimum of any social welfare function. As a result, contrary to Chetty and

[6] This relies to the extensively studied literature on games on networks, see Jackson and Zenou (2014), Bramoullé and Kranton (2016).

Saez (2010), the optimal linear tax rate should account for informal transfers.[7]

I then investigate the design of a non-linear income tax, which enables to better target individuals. The idea is that the government can now reduce tax rates on incomes that are statistically more present in groups in which higher redistribution takes place to encourage more non-distortive redistribution. The standard sufficient statistics formula of Diamond (1998), Piketty (1997) and Saez (2001) pinning down marginal tax rates at the optimum is modified to take into account the impact of marginal tax rates on private transfers. This extra term now highlights an indirect welfare effect that might result, quite surprisingly, in increased or decreased marginal tax rates as compared to a situation where there is no altruism. On the one hand, increasing the tax burden by a small amount for a whole group induces an increase in the transfers made by higher skilled individuals to lower skilled ones within this group. This reduces the welfare cost of raising marginal tax rates on incomes below those of this group, calling for higher marginal tax rates. On the other hand, increasing the tax burden on high-skilled individuals of a group but not on the low-skilled decreases the private transfers made within the group. This acts as an indirect tax on the low-skilled of this group and calls for lower marginal tax rates. Using simulations, I illustrate how the optimal non-linear income tax is affected by different group structures. This allows me to revisit the two cases examined in the linear income tax setting and to show that for certain levels of altruism, tax rates may be affected by more than 10 percentage points.

The flexibility of my framework enables me to explore several potential extensions, in which I relax some of the assumptions of the baseline model and expand it in appropriate dimensions. These applications mainly differ in the structure of information available to the State.

Taxation in developing countries. In many countries and settings, small monetary transfers between individuals are pervasive. In developing countries in particular, total private transfers between individuals represent important fractions of total income. Cox and Jimenez (1990) provide examples of average transfer amounts as percentages of average (global[8]) income in a variety of settings. In rural India, over the years 1975-1983, this share was of 8 %, in 1974 in Kenya and in Malaysia it was of 3 % and 11 % respectively. In Appendix A, I use the World Bank's Living Standard Measurement Surveys (LSMS) in six countries – Burkina Faso, Ghana, Mali, Niger, Tajikistan and Uganda – to present some evidence on the importance of these transfers, as well as on their potential observability and patterns. The years covered by these more recent surveys range from 2009 to 2014.
To the extent that altruistic transfers (see Cox et al. (2006)) are important in this context, my baseline results contribute to the literature on taxation in developing countries by considering how such transfers affect the optimal tax *design* in such a setting.[9]
To better take into account the context of developing countries I enrich my baseline framework and consider different motives for transfers. Three other motives for private transfers have been widely

[7]Note that this is in resonance with Chang and Park (2017) for non-linear taxes. In their paper however, the coauthors consider the optimal design of income taxes in the presence of private insurance markets. Both public and private insurance are thus formal.

[8]That is, taking into account income of households who do not make or receive transfers.

[9]Work on taxation in developing countries, is mainly concerned with building State capacity (see Pomeranz and Vila-Belda (2019) for a comprehensive overview of the research in this field). Results provided in this section may be seen as complementary to this work.

studied: reciprocity (also referred to as quid pro quo, or exchange),[10] risk-sharing[11] and finally, social norms that translate into a kinship tax.[12] Transfers pertaining to risk-sharing relate to shorter-term income fluctuations, whereas kinship taxes and altruistic transfers relate to long term income differences between individuals. Using the World Bank's LSMS, I find that in the countries for which this data is available, half of the transfers are made on a regular basis: this provides evidence for the existence of transfers relating to longer-term income differences. I extend my model to include kinship taxes: agents are now linked through altruism in their direct network, and through kinship taxes with surrounding networks. Kinship taxation now adds two different aspects to the design of optimal income taxation. On the one hand, it could be optimal for (formal) income taxes to crowd out this kinship tax, but this would also imply crowding out altruistic transfers. On the other hand, if at the optimum kinship and State taxes co-exist, then the formal income taxe introduces a double distortion on the effort decision of individuals who are taxed both by their surrounding networks and by the State.

Taxation of families. The taxation literature has extensively studied couple taxation (see Chiappori (1988), Chiappori (1992), Alderman, Chiappori, Haddad, Hoddinott and Kanbur (1995), Kleven, Kreiner and Saez (2009), and more recently Gayle and Shephard (2019), Obermeier (2018)). This paper enables, on the other hand, to address the question of taxation of families with children. Here I relax the assumption that the State does not know the groups. Supposing that the groups are families consisting of (one or two) parent(s) and a number of children, the government can observe groups and tag on the type of group the parents belong to. This may help explain a feature of the French tax system, for example, where families with children do not only receive child-related benefits but also see their marginal tax rate decreased on average.

Related literature. The interaction between private altruistic redistribution and State redistribution has been studied in a series of seminal papers – notably by Barro (1974), Becker (1974), Bergstrom, Blume and Varian (1986), Bernheim and Bagwell (1988) and in an extension by Mercier Ythier (2006). These articles hold neutrality results, whereby under certain conditions, private transfers on the one hand, and exogenous, publicly decided redistribution of wealth[13] on the other hand, achieve the same equilibrium.[14] Thus, when the government seeks to redistribute from rich to poor, it just replaces the pre-existing private transfers. A consequence of this result is that if public redistribution comes with administrative costs, then the role of the State for redistribution is irrelevant or at best limited. There are critical aspects, however, that these articles do not consider: as discussed above, in practice, individuals are not exogenously endowed with an income level[15] and there is an asymmetry of information between the State and individuals. Finally, most of these articles rely on the existence of "chains of operational [i.e., positive] transfers" that link individuals even after the introduction of a tax and thus do not consider that individuals may not all be connected through transfers – and

[10] See Cox and Fafchamps (2007) and De Weerdt, Genicot and Mesnard (2019).

[11] See the extensive work of Townsend starting with Townsend (1994), and also Fafchamps and Gubert (2007), Fafchamps and Lund (2003), Mobarak and Rosenzweig (2012; 2013).

[12] See Jakiela and Ozier (2015), Squires (2016), Baland, Guirkinger and Mali (2011), Carranza, Aletheia, Grosset and Kaur (2018), Boltz, Marazyan and Villar (2019).

[13] Public debt and social security in the case of Barro (1974).

[14] See Mercier Ythier (2006) for a detailed exposure of these results.

[15] None of the above-mentioned articles address this point, except for Bernheim and Bagwell (1988). In their paper, however, this productive effort is observable by the State.

in particular that some individuals neither make nor receive transfers even though they are part of a network of altruism.

The crowd-out effect of public redistribution schemes on private transfers has been empirically measured in various settings. The literature focusing on the elasticity of private transfers to taxes and subsidies in developing countries (e.g. Strupat and Klohn (2018), Heemskerk, Norton and De Dehn (2004)) finds evidence for partial crowding-out. For example, Cox and Jimenez (1992) find that in Peru, social security reduced the amount of private transfers from young to old by approximately 20 percent. The sufficient statistics highlighted in my analysis, however, would require measuring the impact of a society-wide lump-sum transfer by the State on private transfers, or of an increase in the tax rate on a given bracket.

Another perspective taken on by the literature has been the study of the coexistence of formal markets and informal networks. Theoretically, Kranton (1996) finds results that depend on the initial share of reciprocal exchange, and Gagnon and Goyal (2017) find that aggregate welfare, inequality and strength of social ties depend on whether the market and the network are complements or substitutes. In their paper, however, individuals have a choice to participate in the formal market: this cannot be the case here, as taxation applies to all individuals in a State. Finally in an interesting take on the question, Galasso and Profeta (2018) adopt a reverse perspective and seek to understand the impact of public pension schemes on the shape of social norms.

This paper is organized as follows. In the next section I present the formal framework. In Section 3, I characterize the economy with no government intervention, and in Section 4, I derive the optimal linear and non-linear income taxes in this setting. In Section 5, I consider some extensions of this framework. I conclude in Section 6. Appendix A documents patterns of private transfers in a selection of countries. In Appendix B, I detail how I conducted the numerical simulations. Unless otherwise stated, all proofs are in Appendix C.

2 The model

Consider an economy where individuals are endowed with different productivity or skill levels and have private and social preferences. Their social (or altruistic) preferences reflect the fact that they value the private utilities of their peers. This then defines a network of altruism, where any two individuals may be considered as nodes, and where one individual valuing the private utility of another individual implies that a link is present between these two nodes. We assume networks of altruism to be of the form of groups to account, for example, for family, extended family, caste or village structures. Our society thus consists of groups with more or less heterogeneity in the skills of individuals that compose them. Within this setting, individuals choose their productive effort and the transfers they make to their peers in order to maximise their altruistic preferences. Finally, we clarify what structures of information we shall consider, and what situations they may reflect.

Skills and private preferences. Individuals are endowed with a skill or productivity level ω, and have private preferences given by a utility function u. This utility function is increasing in private goods consumption – or after-tax-and-(private)-transfer income (ex-post income) – c. It is decreasing in earnings – or income or ex-ante income – y, thus reflecting the cost of working for the individual. The private utility that an individual of productivity ω derives from c and y is thus denoted by $u(c, y, \omega)$.[16]

We assume that $-\frac{u_y(c,y,\omega)}{u_c(c,y,\omega)}$, the marginal rate of substitution between labor and consumption is decreasing in the individual's skills, i.e. for any pair (c, y) and any ω, ω' with $\omega' > \omega$,

$$-\frac{u_y(c, y, \omega')}{u_c(c, y, \omega')} \leq -\frac{u_y(c, y, \omega)}{u_c(c, y, \omega)} .$$

In other words, individuals with higher ability need less compensation in consumption to work more: this is the Spence-Mirrlees single crossing property.

Altruistic preferences. The altruistic preferences of an individual i may be captured by their social utility, which, in the same spirit as Bourlès et al. (2017), we may write as:

$$U_i(c_i, y_i, \omega_i, \{(c_j, y_j, \omega_j)\}) = \frac{1}{1 + \sum_{j \neq i} \alpha_{ij}} \left\{ u(c_i, y_i, \omega_i) + \sum_{j \neq i} \alpha_{ij} u(c_j, y_j, \omega_j) \right\},$$

with $0 \leq \alpha_{ij} < 1$ the level of altruism i has for j.

Here are a few points that may be worth noting.

We normalize social utility by $1 + \sum_{j \neq i} \alpha_{ij}$ in order to ensure that social utility does not diverge with the number of individuals included in an agent's neighborhood (that is, the number of agents in $C(i)$).[17]

The strength of altruism between i and j α_{ij} being smaller than one implies that for equal consumption levels, individual i always values their own marginal consumption more than they do that of individual j.

The network of altruism A is the adjacency matrix that links social and private utilities. If U is the vector of social utilities and u the vector of private utilities in the society, then the network of altruism is such that $U = Au$. It has a diagonal A_{ii} of $\frac{1}{1 + \sum_{j \neq i} \alpha_{ij}}$, and other entries A_{ij} equal to $\frac{\alpha_{ij}}{1 + \sum_{j \neq i} \alpha_{ij}}$.

We define $C(i)$ to be the neighborhood of individual i – that is the set of individuals towards which i is altruistic (and by convention we include i in $C(i)$). These are the individuals on which they put a

[16] This may also be referred to, as in Pareto (1916), as their *ophelimity*.

[17] This is only one of the multiple ways this normalization could be done. We could also normalize it by the degree of individuals, $\frac{1}{\#C(i)}$ or by the maximal degree in society $\frac{1}{max_i\{\#C(i)\}}$. As stated in the text, this normalization just ensures that social utilities converge when the number of individuals present in an agent's network becomes large. It does not impact our results.

positive weight in their social utility function: $C(i) = \{j \text{ s.t. } \alpha_{ij} > 0\} \cup \{i\}$. We also refer to them as i's peers.

Game. Individuals make (potentially null) transfers to each other, so that consumption is of the following form:

$$c_i = y_i - T(y_i) - \sum_{j \neq i} t_{ij} + \sum_{j \neq i} t_{ji},$$

where t_{ij} are transfers from individual i to individual j and t_{ji} from individual j to individual i (so that by definition t_{ij} and t_{ji} are both non negative), and $T(.)$ is the income tax decided upon by the government. We define the *laissez-faire* as the situation in which there is no government intervention: $T(y) = 0$ for all y.

Individuals then choose their productive effort and the transfers they make at the same time taking the choices of the other individuals in their network as given. That is, productive effort and transfer levels are determined as a Nash Equilibrium of a non-cooperative simultaneous game.

Groups and skill attribution. In this paper, we follow Arrow (1981) and consider individuals to be part of a specific type of network, a network composed of groups.[18] The group structure is useful for the following reasons. First, it allows us to capture altruism within families, extended families, castes, villages, religious or ethnic entities. Second, as will become clear below, it enables to add structure on the level of heterogeneity of skills at the society level on the one hand and at the peer level one the other hand.[19]

When our society is composed of groups, an individual i belongs to the set of neighbors of j if and only if their sets of neighbors are the same. In mathematical terms: $i \in C(j) \iff C(j) = C(i)$. We also consider that within a group C, individuals have the same altruistic links to each other, say α^C.[20] Let $\mathcal{C}$ be the set of groups. Thus, at the society level, the adjacency matrix of altruistic ties may be represented by a block diagonal matrix, whose $\#\{\mathcal{C}\}$ blocks are matrices with a diagonal equal to $\dfrac{1}{1+\alpha^C n^C}$ and all other entries equal to $\dfrac{\alpha^C}{1+\alpha^C n^C}$, $C \in \mathcal{C}$.

Groups have characteristics, and individuals belonging to them have different productivity levels ω. First, individuals are assigned to groups. Then, the process of group characteristic and skill realizations will be as follows. Let $C \in \mathcal{C}$ be a given group. In a first step, the characteristics $\boldsymbol{\theta}^C \in \Theta$ of group C are drawn from a distribution with cumulative distribution function $G(\boldsymbol{\theta})$. $\boldsymbol{\theta}^C$ is a vector, that may consist for example of a mean μ^C and a standard deviation σ^C – and eventually of bounds $\underline{\omega}^C$, $\overline{\omega}^C$. This then determines the cumulative distribution function $H(.|\boldsymbol{\theta}^C)$ of a new distribution. In a second step, the skills ω of individuals in group C are drawn from this distribution $H(\omega|\boldsymbol{\theta}^C)$.

[18] That is, the network of altruism is composed of disjoint networks, each of which is complete. Thus, if an individual belongs to one group, then they do not belong to another group.

[19] Other forms of networks may be considered, but some structure needs to be added in order to be able to draw systematic conclusions. If there were no structure, then both links between individuals and skills would be randomly drawn and uncorrelated.

[20] Altruism α may depend on the size n of the group. The only required assumption is that it converges as n becomes infinitely large.

Let us also define $F(\omega)$ with density $f(\omega) = \int_{\boldsymbol{\theta}} h(\omega|\boldsymbol{\theta})g(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$.

Here, let us highlight four points.

First, the process of group and skill attribution enables us to capture both within and between group heterogeneity. Group characteristics $\boldsymbol{\theta}$ and the distribution of skills given these characteristics, H represent the within group heterogeneity in skills, while G, the distribution function of the characteristics of groups, represents between group heterogeneity. In particular, if G has a Dirac shape, then on average all groups are the same.

Second, and related, even though we model group formation as exogenous, the different $\boldsymbol{\theta}$'s may make the groups in society more or less homogeneous in productivity levels. For example if $\boldsymbol{\theta}$ consists of a mean and a variance, and variances are very small, then groups will tend on average to be very homogeneous in skill levels: this may be a way to account for homophily along the skill dimension.

Third, the definition of $\boldsymbol{\theta}$ may not be so limited. It may be more general, by defining $\boldsymbol{\theta}$ as partly determining the characteristics of the distribution of skills in the group, and partly other characteristics (for example, altruism α or size of groups n). In this case we may write $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$, where for example $\boldsymbol{\theta}_1 = (\mu, \sigma)$ is relevant for the distribution of skills in the group,[21] and $\boldsymbol{\theta}_2 = (\alpha, n)$ determines its other relevant characteristics.

Fourth, note that in what follows we may use an abuse of notation in writing μ^θ, σ^θ, $\underline{\omega}^\theta$ and $\overline{\omega}^\theta$ for the mean, standard deviation, and lower and upper bounds of the distribution of skills in a group of type $\boldsymbol{\theta}$.

Information structure. The individuals know their own skill level, who their peers are (any i knows $C(i)$), and the skill levels of their peers. In the baseline model, the government does not know who the peers of an individual are, and does not know the size (or distribution of sizes) of groups either. Thus, the government only knows $H(\omega|\boldsymbol{\theta})$ and $G(\boldsymbol{\theta})$, as well as the individuals' earnings. This framework may be applied to the case of caste-based groups that span many villages as described in Munshi (*forthcoming*), extended family networks, religious or cultural networks that span many localities.
We refer to $(\omega_i, \boldsymbol{\theta}^{C(i)})$ as an individual i's type in the government's point of view: it is composed of their skill level and the characteristics of the group they belong to.

In some extensions and applications (see Section 5), we consider different structures of information. Indeed, the government may know the groups. This would apply for example to families: the government knows about the link between parents and children. It may know $\boldsymbol{\theta}$, for example if altruism is present at a village level, and the government knows the average skill level in different villages. Finally it may not perfectly observe earnings, as in many developing countries.[22]

[21] Within a group C, skills are then drawn from $H(\omega|\theta_1^C)$.

[22] Even though with the rise of formal employment and third party reporting, some attempts at broadening the tax base to include income taxation are starting to arise, see the recent reform in Uganda, World Bank (2018).

Specification. In what follows, we shall use the following specification for the private utility functions:

$$u(c_i, y_i, \omega_i) = u\left(c_i - k\left(\frac{y_i}{\omega_i}\right)\right),$$

where utility u is increasing and concave and k, which reflects the cost of effort, is increasing and convex. By definition, this cost is thus given in consumption units.

Note that using the concave transformation of a quasi-linear utility function enables us to avoid income effects, and thus to isolate the interaction between taxes and private transfers on a single dimension. This assumption is quite common in the public finance literature (see e.g., Diamond (1998), Saez and Stantcheva (2016), Sachs, Tsyvinski and Werquin (2016), Chetty and Saez (2010), Saez, Slemrod and Giertz (2012)...) and consistent with the empirical evidence, which finds very small income effects (see e.g. Gruber and Saez (2002), Cabannes, Houdré and Landais (2014)).

Examples. In the following, I shall use two running examples in order to illustrate the effects at play. These also show that the group structure presents a pretty flexible framework.
In one example (*Example 1*), there are only two productivity types ω_H and ω_L, so that the important group characteristic is the proportion π of high-skilled individuals in the group. Thus g will refer to the distribution of these proportions. And h will then be discrete, with $h(\omega_H) := \mathbb{P}(\omega = \omega_H) = \pi$, $h(\omega_L) := \mathbb{P}(\omega = \omega_L) = 1 - \pi$.
In the other example (*Example 2*), I shall compare two kinds of societies:
- Society A, where all groups are the same (i.e. $\boldsymbol{\theta}$ is the same for all groups).
- Society B, where there are three kinds of groups. That is, $\boldsymbol{\theta} = \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3\}$. Now g is discrete, with $g(\boldsymbol{\theta}_i) := \mathbb{P}(\boldsymbol{\theta} = \boldsymbol{\theta}_i) = p_i$, such that $p_1 + p_2 + p_3 = 1$. The groups will be more or less homogeneous in skill levels, and with higher or lower skill levels on average.

3 No government intervention

Let us first note that given the structure of the network of altruism, it is never the case that an individual belonging to a group makes a transfer to an individual belonging to another group of the network.

The program of individual i is the following:

$$\max_{\{y_i, t_{ij}, j \neq i\}} u\left(y_i - \sum_{\substack{j \neq i \\ j \in C(i)}} t_{ij} + \sum_{\substack{j \neq i \\ j \in C(i)}} t_{ji} - k\left(\frac{y_i}{\omega_i}\right)\right) + \alpha \sum_{\substack{j \neq i \\ j \in C(i)}} u\left(y_j - \sum_{\substack{k \neq j \\ k \in C(j)}} t_{jk} + \sum_{\substack{k \neq j \\ k \in C(j)}} t_{kj} - k\left(\frac{y_j}{\omega_j}\right)\right)$$

Their first order condition with respect to earnings y_i specifies that at the optimum the marginal rate of substitution between labor and consumption is equal to one:

$$u_c(c_i, y_i, \omega_i) = -u_y(c_i, y_i, \omega_i) \qquad (1)$$

With respect to transfer t_{ij}, it is such that at the optimum, the marginal utility of individual i is at least equal to the marginal utility of their peers, weighted by the strength of altruism:

$$-u_c(c_i, y_i, \omega_i) + \alpha u_c(c_j, y_j, \omega_j) \leq 0 \qquad (2)$$

If the transfer made by i to j is positive, i.e., $t_{ij} > 0$, then the second condition binds.

Moreover, transfers between two individuals cannot flow in both directions: if $t_{ij} > 0$, then $t_{ji} = 0$.[23]

Proposition 1 *A Nash equilibrium exists and is such that within each group C the following characterization can be made.*

General characterization.

(i) The longest path of transfers is of length 1.

(ii) Transfers may only flow from higher skilled to lower skilled individuals.

(iii) Income and consumption allocations are uniquely determined for each skill.

(iv) The sum of net transfers of each individual is unique, but the implementation of transfers is not.

Incomes. *Incomes are determined by the first order condition:* $\frac{1}{\omega}k'\left(\frac{y}{\omega}\right) = 1$.

Transfers.
Define ω_M^C (resp. ω_m^C) to be the largest (resp. smallest) skill level in C, and y_M^C (resp. y_m^C) to be the highest (resp. smallest) equilibrium income in group C.

Necessary and sufficient condition.

There are positive transfers in group C if and only if $u'\left(y_M^C - k\left(\frac{y_M^C}{\omega_M^C}\right)\right) < \alpha u'\left(y_m^C - k\left(\frac{y_m^C}{\omega_m^C}\right)\right)$.

Characterization.

If transfers take place in group C, then there exists a unique pair (Q_m^C, Q_M^C) such that:

All individuals with $y_i - k\left(\frac{y_i}{\omega_i}\right) < Q_m^C$ receive total transfers t_i such that $y_i + t_i - k\left(\frac{y_i}{\omega_i}\right) = Q_m^C$.

[23] Indeed, suppose that at the equilibrium both are positive. Then,

$$u_c(c_i, y_i, \omega_i) = \alpha u_c(c_j, y_j, \omega_j) < u_c(c_j, y_j, \omega_j) \quad \text{and} \quad u_c(c_j, y_j, \omega_j) = \alpha u_c(c_i, y_i, \omega_i) < u_c(c_i, y_i, \omega_i),$$

which is a contradiction.
If both were positive, since altruism $\alpha < 1$ (individuals attach more weight to themselves than to their peers), one individual would have an incentive to deviate and this could not be an equilibrium.

All individuals with $y_i - k\left(\frac{y_i}{\omega_i}\right) > Q_M^C$ *make total transfers* t_i *such that* $y_i - t_i - k\left(\frac{y_i}{\omega_i}\right) = Q_M^C$.
All individuals with $Q_m^C \leq y_i - k\left(\frac{y_i}{\omega_i}\right) \leq Q_M^C$ *neither make nor receive transfers.*

Proposition 1 is similar to Arrow (1981)'s charity game except for the fact that individuals are not assigned individual incomes, but make a productive effort decision that leads to an endogenous earnings distribution in the population.

The first point (i) of Proposition 1 can be put differently: transfers flow directly from donors to recipients. In the terms of Bourlès et al. (2017), they flow through paths of highest altruistic strength. If they did not, then, because individuals attach more weight to themselves than to their peers, there would be a profitable deviation for at least one agent along the path of transfers. In particular, an individual cannot be both a donor and a recipient.

The second point (ii) of Proposition 1 comes from the fact that working is more costly for lower skilled individuals than it is for higher skilled individuals, so that at equilibrium, their income net of their cost of effort is lower than that of higher skilled individuals. Hence, it is never optimal for them to make transfers to more productive individuals. In the same way, it would never make sense for two individuals of the same skill level to make transfers to each other. In particular, if the groups are completely homogeneous, then no transfers take place. Hence, the only kinds of transfers that might take place at equilibrium are transfers from higher productivity individuals to lower productivity ones. Note that this, as mentioned in the introduction, means that these transfers are redistributive.

Effect of an increase in altruism. The higher the altruism level in a group, the lower the inequality level in this group. (The proof is in the Appendix).

Point (iii) of Proposition 1 states that there is a unique equilibrium allocation for each individual; moreover, incomes and consumption are identical for each skill level within a group.

Moreover, since there are no income effects, productive effort decisions, therefore incomes are the same as what they would have been in the absence of altruism. Hence, incomes do no depend on the groups individuals belong to, but their consumption does.[24]

Transfers are then group-specific and determine individuals' consumption levels. Within each group, either there is no transfer at all, or transfers take place and the total sum of net transfers for each productivity level is uniquely determined. However, the implementation of transfers is not unique. For example, suppose we are in a group with two individuals A and B of skill level ω_H, and two individuals C and D of skill level ω_L, such that $\omega_L < \omega_H$. Suppose there are transfers taking place at the equilibrium, and that ω_L is the only benefactor skill level and ω_H is the only donor skill level. Then, A and B each make the same sum of transfers t. This could however take place in an infinity of ways. In particular, A could transfer t to C, and B could do the same to D. Or A could transfer $t/2$ to C and

[24] If there are income effects and private utility functions are separable in consumption and labor, then the general characterization of the equilibrium still carries out even though both income and consumption are then group-specific.

$t/2$ to D, and B the same.

Let us name the condition for transfers to take place in a group C (T):

$$u'\left(y_M^C - k\left(\frac{y_M^C}{\omega_M^C}\right)\right) < \alpha u'\left(y_m^C - k\left(\frac{y_m^C}{\omega_m^C}\right)\right).$$

It is clear from this condition (T) that if α is too low, if skill levels are too close or if u is not concave enough, then no transfers occur in these groups. Indeed, income net of the cost of effort is larger for the most skilled individual than it is for the least skilled one, so that the former's marginal utility is lower than the latter's. For this to carry out when the marginal utility of the lowest skilled individual is weighted by a factor smaller than one, the altruism weight (α) should not be too low, skill levels should be different enough, or the utility functions should have enough curvature.[25]

In particular, it might be that in a group with a high variance in skills, an individual with given skill level makes or receives transfers. Had they been born in a more homogeneous group, they would not have done so. Thus, inequality in consumption net of effort is due both to the skill levels of individuals and the heterogeneity of skill levels in the group to which they belong.

Finally, in this structure of networks, net consumption in equilibrium has a specific form. If transfers take place in a group, then there are three types of agents in this group. There are Donors, Beneficiaries and agents who neither give nor receive (whom we shall call Neutral individuals). Of course, as made explicit by point (i) of Proposition 1, these types cannot overlap. What the characterization in Proposition 1 means, is that all Donors from group C have same final net consumption Q_M^C, all Receivers in group C have same final net consumption Q_m^C, and Neutral individuals consume what they earn.

Q_m^C and Q_M^C are related through the following equality $u'(Q_M^C) = \alpha u'(Q_m^C)$, and defined by

$$\sum_{i \text{ s.t. } y_i - k\left(\frac{y_i}{\omega_i}\right) > Q_M^C} \left(y_i - k\left(\frac{y_i}{\omega_i}\right) - Q_M^C\right) = \sum_{i \text{ s.t. } y_i - k\left(\frac{y_i}{\omega_i}\right) < Q_m^C} \left(Q_m^C - \left(y_i - k\left(\frac{y_i}{\omega_i}\right)\right)\right),$$

which is the exact translation that the sum of transfers made is equal to the sum of transfers received.

This structure of transfers thus gives rise to a specific form of redistribution: lower skilled individuals in a group are guaranteed a minimum utility level, which comes with a ceiling on the maximum utility level attainable by the highest skilled individuals in that group. However, note that this not imply that in practice we would observe consumption patterns in steps. Indeed, consumption itself is still increasing in the skill level: higher productivity individuals still consume more.[26]

[25] Of course, we consider a society where there is at least one $\boldsymbol{\theta}$ such that (T) holds, so that the probability that private transfers take place in the society under consideration is positive.

[26] Indeed, for a Donor i in group C, consumption is equal to $y_i - t_i = Q_M^C + k\left(\frac{y_i}{\omega_i}\right)$. So that if j is another Donor in group C with $\omega_i < \omega_j$, their consumption $y_j - t_j = Q_M^C + k\left(\frac{y_j}{\omega_j}\right)$ is lower, because at equilibrium their productive effort is lower, so $k\left(\frac{y_j}{\omega_j}\right) < k\left(\frac{y_i}{\omega_i}\right)$.

Finally, the equilibrium is generally not Pareto-optimal – highlighting the presence of free-riding. As in Arrow (1981), it is Pareto-optimal if and only if at equilibrium there is only one agent making transfers. Indeed, if many individuals make transfers, they cannot coordinate, and fail to internalize the positive externality that their transfers have on the private utilities of the beneficiaries in their group and thus on their own social utility. Hence, resulting transfers may be too low.[27]

Illustration. *We illustrate the Nash equilibrium allocations for a group of ten individuals whose skills are between $\omega_m = 134$ and $\omega_M = 478$. This implies available incomes (income net of cost of effort) between 448 and 2340. Suppose that their utility function is CRRA with parameter $\eta = 0.9$ and that their cost function is iso-elastic with elasticity $\epsilon = 0.3$. Below are the net incomes and net consumption levels without altruism and with altruism $\alpha = 0.4$.*

(a) Net consumption without altruism
Gini = 0.202

(b) Net consumption with altruism $\alpha = 0.4$
Gini = 0.166

Figure 1: Allocations with and without altruism

This figure illustrates Nash equilibrium allocations with and without altruism. The grey disks represent individuals' net incomes and the red circles their net consumption. When there is no altruism, both coincide: individuals consume what they earn. When there is high enough altruism, the richest individual(s) make transfers to the poorest individual(s). All donors have the same net consumption level, and all receivers have the same net consumption level.

Example 1. *Take a society where there are only two types of skills: $\omega_H > \omega_L$ and where groups differ in their proportion π of high productivity individuals. This example enables us to have comparative statics for when the difference between high and low skill changes (think for example of a parent and children: what is the impact on laissez-faire redistribution of the parent being more skilled), to understand the impact of the share of high skilled on ex-post inequality in the group, and to get some intuitions on how transfers vary with a simultaneous decrease in incomes in a group. Moreover, this allows us to illustrate the neutrality result in a direct way, as well as the changes that would occur to this result in an optimal income taxation setting. Of course however, this example does not highlight the existence of the three types of individuals in a group. Here, either high skilled individuals are Donors and low-skilled ones are Beneficiaries, or else there are no transfers.*

[27] This may be seen with the following example. Suppose there are three individuals in a group, two with high skill ω_H, one with low skill $\omega_L < \omega_H$, and such that $u'\left(y_H - k\left(\frac{y_H}{\omega_H}\right)\right) < \alpha u'\left(y_L - k\left(\frac{y_L}{\omega_L}\right)\right)$. Then, both high-skilled individuals make a transfer of t each to the low-skilled individual, and this transfer is such that $u'\left(y_H - t - k\left(\frac{y_H}{\omega_H}\right)\right) = \alpha u'\left(y_L + 2t - k\left(\frac{y_L}{\omega_L}\right)\right)$. A planner who would intervene afterwards could increase both transfers by a small $0 < \varepsilon \ll 1$, and improve everyone's (social) well-being. Indeed, the high-skilled individuals' social utility would increase by $\varepsilon(1-\alpha)u'\left(y_H - t - k\left(\frac{y_H}{\omega_H}\right)\right) > 0$, and the low-skilled one by $2\varepsilon(1-\alpha^2)u'\left(y_L + 2t - k\left(\frac{y_L}{\omega_L}\right)\right)$.

Take a group with proportion π of high-skilled individuals. The condition (T) for transfers to take place within a group may now be rewritten:

$$u'\left(y_H - k\left(\frac{y_H}{\omega_H}\right)\right) < \alpha u'\left(y_L - k\left(\frac{y_L}{\omega_L}\right)\right).$$

Where y_H and y_L are the same in all groups since there are no income effects.
This is a condition that does not depend on π. What is specific to this setting is thus that if α and utilities are the same in all groups, then if transfers take place in one group, they will take place in all the other groups and vice-versa.

If condition (T) holds, then from Proposition 1 (iii)-(iv), all individuals of high productivity make the same total transfer $t \geq 0$ to individuals of low type. Individuals of low type receive $\frac{\pi}{1-\pi}t$ each.

Let us use isoelastic utility and cost functions where $u\left(c - k\left(\frac{y}{\omega}\right)\right)$ is such that:

$$u(x) = \frac{x^{1-\eta}}{1-\eta} \quad \text{and} \quad k\left(\frac{y}{\omega}\right) = \frac{\left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}.$$

Now incomes are equal to $y = \omega^{1+\epsilon}$. As mentioned above, labour supply does not adjust to transfers, and this stems from the fact that there are no income effects.

Condition (T) can be rewritten as $\alpha > \left(\frac{\omega_L}{\omega_H}\right)^{(1+\epsilon)\eta}$, which clearly highlights that α should not be too small, the ratio of skill levels should be low enough, or η, hence the curvature of utility functions should be high enough.

In that case, in a group C with a proportion π of high-skilled individuals, the transfers made by each high skilled individual each sum to:

$$t = \frac{1}{1+\epsilon}\frac{\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{\alpha^{1/\eta} + \frac{\pi}{1-\pi}},$$

so that consumption levels for high skilled and low skilled agents are respectively,

$$c_H = y_H - t = \frac{\frac{\pi}{1-\pi}\omega_H^{1+\epsilon} + \omega_L^{1+\epsilon}}{(1+\epsilon)(\alpha^{1/\eta} + \frac{\pi}{1-\pi})} \quad \text{and} \quad c_L = y_L + \frac{\pi}{1-\pi}t = \alpha^{1/\eta}\frac{\frac{\pi}{1-\pi}\omega_H^{1+\epsilon} + \omega_L^{1+\epsilon}}{(1+\epsilon)(\alpha^{1/\eta} + \frac{\pi}{1-\pi})}.$$

Here, as stated above, y_H and y_L are the same as if there wasn't any altruism. The inequality in consumption levels, however, is lower than if there wasn't any altruism. As expected, the total transfers t from each high skilled individual decrease when the proportion π of high skilled individuals in the group increases. However, the total transfers received by each low skilled individual, $\frac{\pi}{1-\pi}t$, increase as this proportion π increases. Moreover, the gap between the consumption level of high-skilled individuals and that of low-skilled individuals increases as π increases. Hence, in groups with higher shares of low productivity individuals, high skilled individuals operate more redistribution in absolute terms. In relative terms, the ratio between the two is constant, equal to $\alpha^{-\frac{1}{\eta}}$, so that it is lower the higher

altruism is. On the other hand, consumption levels increase with the share of high-skilled individuals. Hence, the higher the share of high productivity individuals, the higher the consumption levels for all individuals in the group but the larger the difference in consumption levels of high and low-skilled agents.

Finally, if the level of altruism α does not depend on the size of groups n then transfers only depend on respective proportions of high to low skilled agents but not on the size of groups.

Transfers increase with altruism α; the level of consumption of high-skilled individuals decreases with α and that of low-skilled individuals increases with α. So even though altruism is two-sided, in private utility terms, an increase in altruism systematically benefits the low skilled individuals.

Note that this is also a framework that can enable us to think about groups as families. We can take groups as differing in size n, and suppose that groups represent families with one parent (of high-skill ω_H) and $n-1$ children (of low-skill ω_L). Then, $\frac{\pi}{1-\pi} = \frac{1}{n-1}$. The higher the skill of the parent, the higher the children's consumption levels, but the more children there are, the less the parent is able to operate redistribution (even though they increase their transfers).

Finally, suppose agents experience an exogenous lump-sum skill-specific decrease in their income that occurs simultaneously to their productive effort decision. That is, y_H is now equal to $\omega_H^{1+\epsilon} - \varepsilon_H$, and y_L to $\omega_L^{1+\epsilon} - \varepsilon_L$, where ε_H, $\varepsilon_L > 0$. We can of course write $\varepsilon_i = \lambda_i \omega_i^{1+\epsilon}$, for $i = H, L$. The condition for transfers to take place (T_ε) is now

$$\alpha^{1/\eta} > \frac{\omega_L^{1+\epsilon} - \varepsilon_L}{\omega_H^{1+\epsilon} - \varepsilon_H}$$

This condition is easier to satisfy than the initial condition if and only if the share of income lost by the lower skilled individual is higher than that of the higher skilled on: $\lambda_L > \lambda_H$. If this is the case, then transfers still take place, and the transfers made by each high-productivity agent sum to:

$$t = \frac{1}{1+\epsilon} \frac{\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon} + (\varepsilon_L - \alpha^{1/\eta}\varepsilon_H)}{\alpha^{1/\eta} + \frac{\pi}{1-\pi}}.$$

Hence, perhaps counter-intuitively, if altruism is sufficiently low (i.e. $\alpha^{1/\eta} < \varepsilon_L/\varepsilon_H$ – which we note is compatible with (T_ε) only if $\lambda_L > \lambda_H$), then transfers are higher than in the situation where agents have not experienced a lump-sum decrease in their incomes. Indeed, high-skilled agents with a low level of altruism initially make lower transfers to low-skilled agents. Hence, the latter's consumption level is initially lower. Given that utilities are concave, a decrease in their income thus marginally weighs more than if their consumption was initially higher. This can thus result in higher transfers. Note however that low-skilled individuals' consumption level (as opposed to the transfers they receive) is still lower than if altruism was higher (transfers decrease less with an increase in ε_H, the lump-sum decrease undergone by high-skilled agents).
If however, high productivity agents experience a decrease in their income that is proportionately higher (or equal) than that of the low productivity agents (i.e. $\lambda_H \geq \lambda_L$), then no counter-intuitive result arises: transfers are lower.

This also enables us to illustrate one form of the neutrality result. If an omniscient Social Planner levies ε from all high-killed individuals in a group and gives $\frac{\pi}{1-\pi}\varepsilon$ to all low-skilled ones, then final consumption levels are unchanged. It is immediately clear from this example that being able to observe individuals' types is key for the government to be able to operate such redistribution. In practice, this information is not available to the government, and redistribution operated by the State is thus distortive.

Both Proposition 1 and Example 1 enable us to see that altruism reduces inequality, but only at the intra-group level, and more or less so depending on heterogeneity in skills and altruism levels within groups, and finally on how representative of society groups are. Beyond the need to raise revenue, the government might thus also have scope to operate redistribution. In the following we analyse the design of optimal income taxes in such a context.

4 Optimal income taxation

The government is paternalistic. It chooses to maximize a function of private utilities:

$$SWF = \int z_i \Phi(u^i)\, \mathrm{d}\nu(i)$$

where z_i are (arbitrary) Pareto weights, Φ is increasing and concave, and $d\nu(i)$ is the distribution of individuals.[28]

Moreover, the government sets its tax ex-ante (before observing the realization of groups and skills in the society in question), and knows the game that individuals play. The tax is implemented simultaneously to the game played between the agents.

In the baseline model, as specified in Section 2, we suppose that the government can observe neither group characteristics nor skills, and does not know to which group an individual belongs. We also suppose that either it assumes that groups are large enough or it has no information on the distribution of the size of groups. Thus, it reasons by considering representative groups. It only attaches more weight to a group if its characteristics are more prevalent in the society, but does not do so if the group is larger. In the government's perspective, an agent's (unknown) type is thus given by their skill level and the characteristics of the group they belong to: $(\omega, \boldsymbol{\theta})$.[29]

4.1 Optimal linear income taxation

When setting a linear income tax, the government uses a linear tax rate $\tau \in [0, 1]$ on incomes to finance a demogrant R, and potentially an additional exogenous non-transfer spending B. The demogrant is

[28] Another normative objective for the government would be to maximize a function of social utilities, SWF_s, where $SWF_s = \int z_i \Phi(U^i)\, \mathrm{d}\nu(i)$. In the following, however, we consider only a paternalistic social planner. This enables us to avoid reaching illiberal conclusions, as discussed in Sen (1970), Fleurbaey and Maniquet (2011), Fleurbaey and Maniquet (2018) and very clearly summed up in Bierbrauer (2019).

[29] Note that in this part of the model $\boldsymbol{\theta}$ cannot include the size of the group n.

used to finance a lump-sum redistribution of the tax proceeds to the individuals in the population.

The general characterization results of Proposition 1 carry out. We are still in a situation where at equilibrium, in each group, either there are no transfers taking place, or individuals with ability above a certain threshold make transfers to individuals below a certain threshold. The reasoning follows very closely that of the situation with no government intervention. Moreover, we still have that incomes are only skill but not group dependent. Including income effects would call for multidimensional screening such as in Rothschild and Scheuer (2014), Rochet and Choné (1998).

The Social Planner thus seeks to maximize the following function with respect to the tax rate τ and the demogrant R:

$$\int_{\boldsymbol{\theta}} \int_{\omega} z(\omega, \boldsymbol{\theta}) \Phi \left(u \left((1-\tau) y(\omega, \tau) - t(\omega, \boldsymbol{\theta}, \tau, R) + R - k \left(\frac{y(\omega, \tau)}{\omega} \right) \right) \right) h(\omega | \boldsymbol{\theta}) g(\boldsymbol{\theta}) \, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta},$$

under the (government) budget constraint

$$R = \tau \int_{\omega} y(\omega, \tau) \, f(\omega) \, \mathrm{d}\omega - B,$$

and taking the game played by the agents into account.

For ease of exposition, and since we know that the sum of transfers made or received by individuals is uniquely determined, we can express t to be the net sum of transfers for each individual: it is negative if the individual is a beneficiary, positive if they are a donor and null if they are neither (i.e., neutral). In a first step, the government establishes how agents react to a given tax rate and demogrant and solves the Nash equilibrium of the game of agents in each group by extending it to a continuum of individuals. This is what justifies the notation $t(\omega, \theta, \tau, R)$: from the government's point of view, an individual's net transfers depend on their productivity, the characteristics of their group, the tax rate and the demogrant.

As stated at the beginning of the section, $\Phi(.)$ is an non-decreasing and (potentially weakly) concave function.

The Pareto weights $z(\omega, \boldsymbol{\theta})$ may depend on the individual's productivity level as well as on the characteristics of the group they belong to.

Finally, we define aggregate earnings $\mathcal{Y}(\tau) := \int_{\omega} y(\omega, \tau) \, f(\omega) \, \mathrm{d}\omega$, which enables us to rewrite the demogrant R as $R = \tau \mathcal{Y}(\tau) - B$.

We name this Problem (I).

Proposition 2 ***Optimal linear income tax rate.***
If it is positive, the optimal linear tax rate τ^* is a solution of the fixed-point problem:

$$\frac{\tau}{1-\tau} = \left(1 - \overline{\phi} - \psi\right) \frac{1}{e}.$$

Where the terms $\overline{\phi}$, e and ψ are defined as follows:

$$\overline{\phi} = \frac{\int_{\boldsymbol{\theta}} \int_{\omega} z(\omega, \boldsymbol{\theta}) \Phi'(u(\omega, \boldsymbol{\theta}, \tau)) u'(\omega, \boldsymbol{\theta}, \tau) y(\omega, \tau)\, h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}}{\mathcal{Y}(\tau) \int_{\boldsymbol{\theta}} \int_{\omega} z(\omega, \boldsymbol{\theta}) \Phi'(u(\omega, \boldsymbol{\theta}, \tau)) u'(\omega, \boldsymbol{\theta}, \tau)\, h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}},$$

$$e = \frac{1-\tau}{\mathcal{Y}} \frac{\mathrm{d}\mathcal{Y}}{\mathrm{d}(1-\tau)},$$

$$\psi = \frac{\int_{\boldsymbol{\theta}} \int_{\omega} z(\omega, \boldsymbol{\theta}) \Phi'(u(\omega, \boldsymbol{\theta}, \tau)) u'(\omega, \boldsymbol{\theta}, \tau) \frac{\mathrm{d}t}{\mathrm{d}\tau}(\omega, \boldsymbol{\theta}, \tau)\, h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}}{\mathcal{Y}(\tau) \int_{\boldsymbol{\theta}} \int_{\omega} z(\omega, \boldsymbol{\theta}) \Phi'(u(\omega, \boldsymbol{\theta}, \tau)) u'(\omega, \boldsymbol{\theta}, \tau)\, h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}}.$$

First note that the optimal tax rate is negatively correlated with these three terms, and second that all three terms depend on the tax rate τ.

The first term $\overline{\phi}$ is the usual average normalized social marginal welfare weight weighted by (ex-ante) incomes. This is inversely related to government redistributive preferences. The higher it is, the lower the tax rate is.

The second term is the elasticity of aggregate earnings with respect to the net-of-tax rate. This translates the efficiency cost of taxation. The higher e is, the higher the distortion of productive effort induced by the tax, and the lower the tax rate is.

The last term is the new factor that the government should take into account: it is the ratio of the expectation of the derivative of transfers with respect to the tax rate weighted by the normalized social marginal welfare weights attached to each individual, over aggregate earnings. The higher it is, the lower the tax rate should be. In order to learn more from it, we require the sign of: $-\int_{\underline{\omega}^{\theta}}^{\omega_m^{\theta}(\tau,R)} \frac{\mathrm{d}t}{\mathrm{d}\tau}(\omega, \theta, \tau, R(\tau))\, h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega = \int_{\omega_M^{\theta}(\tau,R)}^{\overline{\omega}^{\theta}} \frac{\mathrm{d}t}{\mathrm{d}\tau}(\omega, \theta, \tau, R(\tau))\, h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega.$

Lemma 1 ***Variation of transfers with R: $\frac{\partial t}{\partial R}$.***
(i) If the utility function u exhibits constant absolute risk aversion, then an increase in the lump-sum transfer R leaves transfers unchanged.
(ii) If the utility function u exhibits decreasing absolute risk aversion, then an increase in the lump-sum transfer R decreases the proportions of donors and beneficiaries and crowds out private transfers.
(iii) If the utility function u exhibits increasing absolute risk aversion, then an increase in the lump-sum transfer R increases the proportions of donors and beneficiaries and crowds in private transfers.

The variations uncovered here are consistent with the shape of these utility functions.

Let us note that both CRRA and HARA utility functions exhibit property (ii) of Lemma 1. Functions that have a negative third derivative exhibit property (iii) of Lemma 1.

In the rest of the paper, we shall use the properties in (i) and (ii) of Lemma 1, as these are the most consistent with empirical estimates.

Denoting by t^B transfers received and by t^D transfers made, the total transfers received and made in a group of characteristics $\boldsymbol{\theta}$ are respectively equal to:

$$\int_{\underline{\omega}^\theta}^{\omega_m^\theta(\tau,R)} t^B(\omega,\theta,\tau,R)\, h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \quad \text{and} \quad \int_{\omega_M^\theta(\tau,R)}^{\overline{\omega}^\theta} t^D(\omega,\theta,\tau,R)\, h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega.$$

Lemma 2 ***Variation of transfers with τ: $\dfrac{\partial t}{\partial \tau}$.***
If the utility function u exhibits constant or increasing relative risk aversion then an increase in the tax rate τ decreases either the proportion of donors, or the proportion of beneficiaries, or both: it crowds out total private transfers.

Let us note that CARA, CRRA, HARA and utility functions with a negative third derivative (that are hence IARA) exhibit one of these properties.

Moreover, we have that the effect of an increase in the tax rate can be decomposed into two:

$$\frac{\mathrm{d}t^B}{\mathrm{d}\tau} = \frac{\partial t^B}{\partial \tau} + \frac{\partial R}{\partial \tau}\frac{\partial t^B}{\partial R} \quad \text{and} \quad \frac{\mathrm{d}t^D}{\mathrm{d}\tau} = \frac{\partial t^D}{\partial \tau} + \frac{\partial R}{\partial \tau}\frac{\partial t^D}{\partial R}$$

Hence, for tax rates such that $\frac{\partial R}{\partial \tau} > 0$, the linear income tax rate crowds out total transfers if the utility function exhibits constant or decreasing absolute risk aversion and constant or increasing relative risk aversion. This is the case for the usual utility functions, such CARA, CRRA and HARA utility functions.[30] Note also that at the optimal tax rate, $\dfrac{\partial R}{\partial \tau} > 0$.

This now enables us to further characterize our optimal linear income tax rate.

Comparison with the standard linear tax rate. The standard formula for the optimal linear income tax rate (see Piketty and Saez (2013)) is $\dfrac{\tau_{NA}}{1+\tau_{NA}} = \left(1-\overline{\phi}_{NA}\right)\dfrac{1}{e_{NA}}$, where NA stands for "no altruism".

If the optimal tax rate in the presence of altruism is such that all transfers are crowded out, then ψ is equal to 0 and individuals' consumption levels do not depend on the group they belong to. So if Pareto weights z do not depend on group characteristics, then the tax rate τ^* is the same as τ_{NA}.

[30] If the utility function is IARA, however, then the effect is ambiguous.

Hence if transfers decrease with the tax rate, and if at the optimum there is no complete crowding out of transfers, then τ_{NA} represents an upper bound for the optimal tax rate in the presence of altruistic transfers.

Moreover, the demogrant $R = \tau\mathcal{Y}(\tau)$ is equal to $-B$ at tax rates of 0 and 1, is first increasing and then decreasing, and its derivative changes signs at the revenue maximizing tax rate $\underline{\tau}$, $\dfrac{1}{1+e(\underline{\tau})}$. At both optima (the altruistic and non altruistic ones), its derivative is positive. Thus, since $\tau^* \leq \tau_{NA}$, $R(\tau^*) \leq R(\tau_{NA})$. When there are no private transfers, the government uses a higher tax rate to redistribute more lump-sum to individuals.

Rawlsian social planner. In the absence of private transfers, the tax rate chosen by a Rawlsian social planner who only associates weight to the poorest individual in society is the revenue maximizing one, $\underline{\tau}$. Indeed, if the society is not perfectly equal (if F does not have a Dirac shape), then $\overline{\phi}_{NA}$ is equal to 0.[31] However, in the presence of altruism, this average social marginal welfare weight weighted by incomes is not necessarily equal to 0. For example, if groups all have the same characteristics, and if the optimal tax rate does not completely crowd out private transfers, then the objective of the government is to maximize $Q_m(\tau)$, the minimum guaranteed consumption for poorer individuals through transfers. It is straightforward to show that Q_m is decreasing at least from the revenue maximizing tax rate $\underline{\tau}$ onwards. Hence, the optimal tax rate of a Rawlsian social planner in an altruistic society is potentially lower than $\underline{\tau}$.

The weighted derivative of transfers with respect to the tax rate ψ. Suppose that Pareto weights are homogeneous in the population.[32] We know from Proposition 1 that if there are transfers taking place, all donors have the same net consumption and all beneficiaries have the same net consumption. Thus, using the fact that within a group total transfers received is equal to total transfers made, for each $\boldsymbol{\theta}$,

$$\int_\omega \Phi'(u(\omega,\boldsymbol{\theta},\tau))u'(\omega,\boldsymbol{\theta},\tau)\frac{\mathrm{d}t}{\mathrm{d}\tau}(\omega,\boldsymbol{\theta},\tau)\,h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega$$
$$= (\Phi'(u(Q_M^\theta))u'(Q_M^\theta) - \Phi'(u(Q_m^\theta))u'(Q_m^\theta))\int_{\omega \text{ donor}} \frac{\mathrm{d}t}{\mathrm{d}\tau}(\omega,\boldsymbol{\theta},\tau)\,h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega.$$

Now since $Q_M^\theta > Q_m^\theta$ and Φ and u are concave and increasing, $\Phi'(u(Q_M^\theta))u'(Q_M^\theta) < \Phi'(u(Q_m^\theta))u'(Q_m^\theta)$. So if transfers decrease with the tax rate (which from Lemma 1 and Lemma 2 we know is the case for standard utility functions), then $\psi > 0$, thus decreasing the optimal tax rate τ^*.

Moreover, the more concave Φ is, the greater ψ is, and hence the lower the tax rate is.

This highlights two opposing forces between $\overline{\phi}$ and ψ. Indeed, the more concave Φ is, the more weight the government attaches to poorer[33] individuals, and for a given distribution of after-tax net consumption levels, the lower $\overline{\phi}$ is and the higher ψ is.

[31] As there is a mass 0 of individuals with productivity $\underline{\omega}$.
[32] That is, for all skill levels ω, for all vectors of group characteristics $\boldsymbol{\theta}$, $z(\omega,\boldsymbol{\theta}) = z$.
[33] Poorer in net consumption levels.

Thus, by taking into consideration the partial crowding out of transfers, the government may take advantage of the fact that there is already some redistribution occurring at a private level – a redistribution that does not distort labor supply incentives – and thus decrease the marginal tax rate used to redistribute at the society level – thus also reducing the amount of distortion created by the tax: private transfers, if they are high enough, help the government achieve its equity goal at a lower efficiency cost.

Example 1. (Linear income tax) *Let us follow-up with our example from the previous section. This will give us a sense of when the social planner will find it efficient to encourage partial redistribution through private transfers while at the same redistributing at the society level.*

In this example, we suppose that altruism, elasticities and coefficients of relative risk aversion are the same in all groups.

Remember we use the following specifications for our utility and cost of effort functions:

$$u\left(c - k\left(\frac{y}{\omega}\right)\right) \quad \textit{such that} \quad u(x) = \frac{x^{1-\eta}}{1-\eta} \quad \textit{and} \quad k\left(\frac{y}{\omega}\right) = \frac{\left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}.$$

Here, incomes of high and low ability individuals are the following:

$$y_H = (1-\tau)^{\epsilon}\omega_H^{1+\epsilon} \quad \textit{and} \quad y_L = (1-\tau)^{\epsilon}\omega_L^{1+\epsilon}.$$

Again, let us note that labor supply does not depend on transfers. It is the same as it would have been without any altruism in the society. The linear income tax, however, now distorts the productive effort decisions: the higher it is, the lower the earnings.

Suppose the government has no exogenous government budget requirement ($B = 0$): the tax is purely redistributive. Then, there are still transfers in equilibrium if and only if $\tau < \overline{\tau}$,

$$\textit{where } \overline{\tau} = \frac{\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{(\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) + (1-\alpha^{1/\eta})(1+\epsilon)(\mu_\pi\omega_H^{1+\epsilon} + (1-\mu_\pi)\omega_L^{1+\epsilon})},$$

with $\mu_\pi := \mathbb{E}(\pi)$, the average of π's (the proportion of high skilled individuals in the various groups) over the population.

The necessary and sufficient condition for transfers to take place in the laissez-faire economy

$$\alpha^{1/\eta} > \left(\frac{\omega_L}{\omega_H}\right)^{1+\epsilon},$$

thus remains a necessary condition.

This enables us to notice that $\overline{\tau}$ is a function of exogenous parameters, and does not depend on specific

values of π, but just on their average. Hence, the equilibria are of the same type in all groups of the network: at equilibrium, either there are transfers taking place in all groups (except the ones with $\pi = 0$ or 1, i.e., with only high-skilled or low skilled individuals) or no transfers are taking place. Moreover, $\overline{\tau}$ is decreasing in μ_π: the higher the average share of low skilled individuals, the more likely it is that the optimal tax rate will encourage some private transfers. Indeed, as we saw in the laissez-faire situation, the same comparative static can be made here: groups with lower shares of high skilled individuals operate more redistribution between high and low skilled individuals. If there are enough such groups, the social planner reaches higher social welfare by partially encouraging private redistribution. Moreover, this cut-off tax rate is also increasing in the altruism level of society.

If the tax rate τ is greater than the threshold rate $\overline{\tau}$, in a group with a proportion π of high-skilled individuals, transfers made by high-skilled individuals are:

$$t = \frac{\frac{(1-\tau)^{1+\epsilon}}{1+\epsilon}(\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) - R(\tau)(1-\alpha^{1/\eta})}{\alpha^{1/\eta} + \frac{\pi}{1-\pi}},$$

where $R(\tau) = \tau(1-\tau)^\epsilon(\mu_\pi\omega_H^{1+\epsilon} + (1-\mu_\pi)\omega_L^{1+\epsilon})$.

This enables us to notice that transfers will be lower than in the laissez-faire case for two reasons. On the one hand, the term $\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}$ is now weighted by $\dfrac{(1-\tau)^{1+\epsilon}}{1+\epsilon} < 1$: the linear tax rate partially crowds out transfers, as high ability individuals now have lower disposable income. On the other hand, since $\alpha^{1/\eta} < 1$, lump-sum redistribution $R(\tau)$ decreases the incentives to make transfers, as the government is already partially taking care of redistribution.

Now, as in a non altruistic model, it cannot be that at the optimum, $\dfrac{\partial R}{\partial \tau} < 0$. Hence and at the optimum, $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} \leq 0$. This is in line with the examples we considered in the laissez-faire part of Example 1. Indeed, in this case, as long as R is positive, the proportion of disposable income lost by the higher skilled individuals is higher than that lost by the low-skilled individual.[34]

Finally, we may partly characterize the optimum, where from the fact that the demogrant is non-decreasing at the optimum, $\dfrac{\mathrm{d}R}{\mathrm{d}\tau} \geq 0$, we know that $\tau^* \leq \dfrac{1}{1+\epsilon}$, the revenue maximizing rate. We will have two cases:

If $\overline{\tau} > \dfrac{1}{1+\epsilon}$, then the optimal tax rate τ^* is such that transfers have not been completely crowded-out at the optimum, and the rate has to be adjusted downwards so as to encourage these to take place. Note that this is more likely the lower μ_π is.

If on the other hand $\overline{\tau} \leq \dfrac{1}{1+\epsilon}$, then the optimal tax rate might be in $[0, \overline{\tau})$ and hence encourage transfers or in $[\overline{\tau}, \dfrac{1}{1+\epsilon}]$ and hence crowd out transfers. This will ultimately depend on the distribution of group shares of high-skilled individuals in the society (more specifically their mean). This is summarized in the figure below.

[34] With the notations of Example 1 in Section 3, this is roughly saying that $\lambda_L < \lambda_H$.

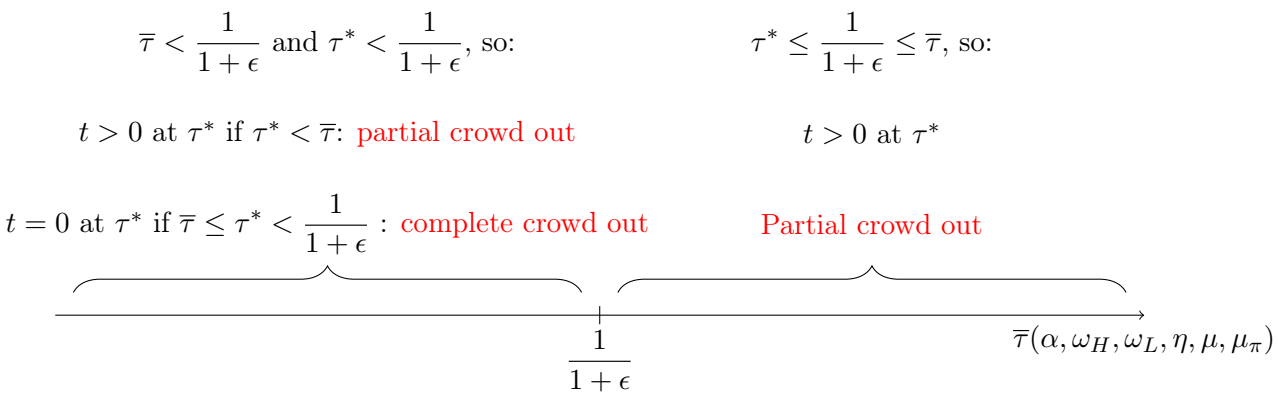


Figure 2: Crowding-out at the optimum

From this figure it is clear that $\overline{\tau}$ being large enough (i.e., greater than the revenue maximizing rate $\frac{1}{1+\epsilon}$) is a sufficient condition for transfers and taxes to coexist at the optimum.

Example 2. (Linear income Tax) *In this example, let us consider a utilitarian social planner (so that $\Phi = id$, and Pareto weights are homogeneous) with no exogenous government expenditure ($B = 0$). Suppose all groups have the same level of altruism (which we may refer to as a society level of altruism), say* 0.6.

First case: Society A. The society is composed of groups with skills drawn from a $\mathcal{N}(400, 100)$. Then, the optimal linear income tax when there is no altruism is equal to 20%. *When the government adjusts the tax rate to take into account altruistic transfers, however, the optimal linear income tax rate is of* 5%. *There is a partial crowd out of transfers but the optimal tax rate is substantially decreased. Thus, the government chooses a tax rate that distorts productive effort decisions less. It is worth noting here that in the presence of a large number of individuals in each group, the individuals' objective function coincides the government's. However, as mentioned in the previous section, since the equilibrium is a non-cooperative one, the optimum that they achieve is possibly lower than the one the government, by centralizing the decision, can achieve. Hence the positive tax rate.*

Second case: Society B. The society is composed of three types of groups with equiprobable realizations (i.e., $p_1 = p_2 = p_3 = 1/3$): skills are drawn either from a $\mathcal{N}(400, 100)$ or a $\mathcal{N}(200, 30)$ or a $\mathcal{N}(600, 30)$.[35] *In this society, there are two relatively homogeneous groups (the "poor" and the "rich") and one heterogeneous group (comprising "middle income" individuals). The optimal tax rate with no altruism would have been of* 35%. *Taking altruistic transfers into account it is of* 34%. *There is much less adjustment of the optimal linear income tax rate and greater crowding out of private transfers because of the gain from redistributing from very rich to very poor.*

This second case shows that there might be welfare gains if the government were better able to target some groups: either by tagging if it knows the characteristics of groups ($\boldsymbol{\theta}$), or by introducing a non-linear income tax.

[35] It is understood here that these distributions are truncated at 0.

Tagging. The government knows the characteristics of groups but not individual productivity levels.

Tagging has been extensively studied (see e.g., Akerlof (1978), Immonen, Kanbur, Keen and Tuomala (1998), Mankiw and Weinzierl (2010), Cremer, Gahvari and Lozachmeur (2010)) and relies on the government conditioning taxes on observable characteristics of individuals that affect their consumption level other than their productivity.[36] In this case, the government may condition taxes on group characteristics. Here, we still consider groups are large enough or the government has no information about sizes. The idea here is that *ex-ante* the government knows the characteristics of villages or communities for example, but does not know how many individuals there will be in them.

The government can now condition the tax rate and demogrant on group characteristics. Its problem (which we name Problem (II)) is thus to maximize with respect to $\tau(\theta)$ and $R(\theta)$:

$$\int_{\boldsymbol{\theta}} \int_{\omega} z(\omega, \boldsymbol{\theta}) \Phi \left(u \left((1 - \tau(\theta)) y(\omega, \tau(\theta)) - t(\omega, \boldsymbol{\theta}, \tau(\theta)) + R(\theta) - k \left(\frac{y(\omega, \tau(\theta))}{\omega} \right) \right) \right) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta}) \, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$$

Under the constraint that

$$\int_{\theta} R(\theta) \, g(\theta) \, \mathrm{d}\theta = \int_{\theta} \tau(\theta) \underbrace{\int_{\omega} y(\omega, \tau) \, h(\omega|\theta) \, \mathrm{d}\omega}_{\mathcal{Y}(\tau(\theta))} \, g(\theta) \, \mathrm{d}\theta - B$$

and taking into account the individuals' reactions.

Adding the constraint that for all $\boldsymbol{\theta}$, $\tau(\theta) = \tau$ and $R(\theta) = R$ brings us back to Problem (I). Thus, welfare will be higher in this case than in the above one. This does not come as a surprise, as the government has more instruments in this setting.

The government may now adjust tax rates to the level of redistribution taking place in the group. Lump-sum redistristrbution (through the demogrant) then allows redistribution from groups that are richer on average to groups that are poorer on average.

Revisiting the second case of the above Example 2, setting a tax rate of 5% *on the middle income group and redistributing the tax proceeds gathered from this tax to the same group, while setting tax rates of* 0% *on our poor and rich groups and redistributing* 54 *lump-sum from the rich to the poor group yields a higher social welfare level.*

However, the characteristics of the groups are generally not known to the government – and this is the setting of our baseline framework. In that case, it has to rely on non-linear income taxes, which will be the object of the next section.

[36] Note that these characteristics may not necessarily be *immutable*, see Piketty and Saez (2013).

4.2 Optimal non-linear income taxation

In this section, the Social Planner may set a non-linear income tax $T(.)$ that maximizes the following objective:

$$\int_{\boldsymbol{\theta}} \int_{\omega} z(\omega, \boldsymbol{\theta}) \Phi \left(u \left(y(\omega) - T(y(\omega)) - t(\omega, \theta, T) - k \left(\frac{y(\omega)}{\omega} \right) \right) \right) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta}) \, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$$

subject to the government budget constraint:

$$\int_{\omega} T(y(\omega)) \, f(\omega) \, \mathrm{d}\omega \geq B.$$

(and as usual, taking individuals' responses into account).

In order to provide a tractable solution to this problem (which we shall refer to as Problem (III)), I use the perturbation method as in Saez (2001), Piketty (1997), Golosov, Tsyvinski and Werquin (2014), Bierbrauer and Boyer (2018), Jacquet and Lehmann (2017). Note however, that this provides us with necessary but not sufficient conditions.

In order for the tax perturbation to be sufficient, we need the following assumptions, from Assumption 2 in Jacquet and Lehmann (2017):

Assumption. Under the following assumptions, the tax perturbation method yields sufficient conditions:
i) The tax function $T(.)$ is twice differentiable.
ii) For all ω, the second-order condition holds strictly.
iii) For all ω, the function $y \mapsto u\left(y - T(y) - t - k\left(\frac{y}{\omega}\right)\right)$ admits a unique global maximum over $\mathbb{R}+$.

Tax perturbation. Starting from an initial tax schedule $T(.)$, the idea is to perturb the following objective function by a small τ on a small interval $[y_0, y_0 + dy_0]$:

$$\mathcal{L} = \int_{\boldsymbol{\theta}} \int_{\omega} z(\omega, \boldsymbol{\theta}) \Phi \left(u \left(y(\omega) - T(y(\omega)) - t(\omega, \theta) - k \left(\frac{y(\omega)}{\omega} \right) \right) \right) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta}) \, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta} + \lambda \int_{\omega} T(y(\omega)) \, f(\omega) \, \mathrm{d}\omega.$$

More specifically, we perturb the initial tax schedule such that it is replaced by a new schedule that takes the following form:

$$T_{\tau}(y) = \begin{cases} T(y) & \text{if} \quad y \leq y_0 \\ T(y) + \tau(y - y_0) & \text{if} \quad y_0 \leq y \leq y_0 + dy_0 \\ T(y) + \tau dy_0 & \text{if} \quad y \geq y_0 + dy_0 \end{cases}$$

This may be illustrated as follows:

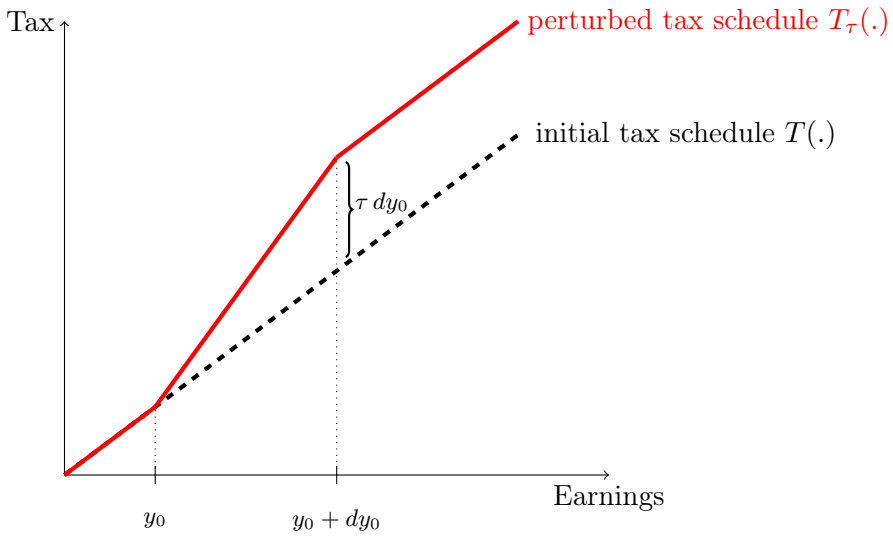


Figure 3: Tax perturbation

Let us note that λ is the marginal cost of public funds and $\phi(\omega, \boldsymbol{\theta}) = \dfrac{z(\omega, \boldsymbol{\theta})\Phi'(u(\omega, \boldsymbol{\theta}))u'(\omega, \boldsymbol{\theta})}{\lambda}$ is the marginal social welfare weight of individual of type $(\omega, \boldsymbol{\theta})$.[37]

The incentive compatibility constraints are the first order and second order conditions determining the income choice of individuals. The first order condition is the following:

$$\frac{1}{\omega}k'\left(\frac{y}{\omega}\right) = 1 - T'(y)$$

Using the envelope theorem, we establish that the perturbation yields the usual three effects (mechanical, welfare and behavioral), but there is now a fourth effect, which arises if transfers still take place at the optimum. This effect is due to the indirect effect that taxes have on transfers and thus on welfare.

Let us refer to the density of earnings at $T(.)$ as $f_y(.)$ and to their cumulative distribution function as $F_y(.)$. As in Saez (2001), let us use the notation $f_y^*(\tilde{y})$ for the density of $\tilde{y}$ if $T(.)$ were linearized at $\tilde{y}$.[38]

The mechanical effect $dM = dy_0 d\tau(1 - F_y(y_0))$ translates the extra revenue that the State gathers thanks to a small uniform increase in tax liabilities for incomes over y_0.

The direct welfare effect $dDW = -dy_0 d\tau \int_{y_0}^{\infty} \phi(y) f_y^*(y)\, \mathrm{d}y$ accounts for the loss in welfare created by this increase in tax liabilities. This direct loss in welfare only impacts individuals with earnings greater or equal to y_0.

The behavioral effect $dB = -dy_0 d\tau e(y_0) y_0 \dfrac{T'(y_0)}{1 - T'(y_0)} f_y^*(y_0)$ accounts for the efficiency loss of increasing marginal tax rates: it quantifies the impact of the increase in marginal tax rates on productive effort. $e(y)$ is the elasticity of earnings with respect to the net-of-tax rate at income level y. Note here

[37] Which implies that we may define the average marginal social welfare weight of individuals with income y as $\phi(y) = \int_{\boldsymbol{\theta}} \phi(\omega, \boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$ for ω such that $y(\omega) = y$.

[38] Refered to in Saez (2001) as the *virtual* density function.

that since there are no income effects, this effect only arises for individuals at y_0.

The additional effect comes from the reaction of transfer decisions to an increase in the tax rate at y_0. In order to derive it, it is important to clarify the notation of t. As in the linear case this notation refers to the net transfer: it is positive for donors, negative for beneficiaries and null for neutral individuals. It thus depends on the skill ω of the individual, but also on the distribution of skills in the group, $\boldsymbol{\theta}$, and on the policy in place, $T(.)$. Thus, for a given policy, it depends on the individual's available income $Y(\omega) = y(\omega) - T(y(\omega)) - k\left(\frac{y(\omega)}{\omega}\right)$ (where $y(\omega)$ depends on the policy in place, $T(.)$ through the individual's first order condition), and on the incomes net of effort of all individuals in the group.
Now for all individuals of productivity at least ω_0, the perturbation has the following effect:

$$\frac{\mathrm{d}Y}{\mathrm{d}\tau} = -dy_0.^{39}$$

We then define $\delta_0 t$ to be such that the effect on t may be written as follows: $\frac{\mathrm{d}t}{\mathrm{d}\tau} = -dy_0 \times \delta_0 t$. Hence, $\delta_0 t$ is the variation in t induced by an infinitesimal increase in all its components from $Y(\omega_0)$ onwards. Thus our last term is the following:

$$dIW = dy_0 d\tau \int_{\boldsymbol{\theta}\,\mathrm{s.t.}\,\omega_0 \leq \overline{\omega}^\theta} \int_\omega \phi(\omega, \boldsymbol{\theta}) \times \delta_0 t \times\ h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$$

where ω_0 is such that $y(\omega_0) = y_0$ under tax schedule $T(.)$.
We shall examine this term more closely in the following paragraphs. This indirect welfare effect is composed both of spillover and dampening or amplifying effects.

A necessary condition for the tax schedule to be optimal is then that $dM + dDW + dIW + dB = 0$ at y_0, which leads us to the following characterization.

Proposition 3 ***Optimal non-linear income tax rates.***
The optimal non-linear income tax rates are such that for each y_0:

$$\frac{T'(y_0)}{1 - T'(y_0)} = \left(\int_{y_0}^{\infty} (1 - \phi(y)) \frac{f_y(y)}{1 - F_y(y_0)}\, \mathrm{d}y + \frac{IW_0}{1 - F_y(y_0)} \right) \times \frac{1 - F_y(y_0)}{y_0 f_y^*(y_0)} \times \frac{1}{e(y_0)}$$

$$\textit{where } IW_0 = \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \leq \overline{\omega}^\theta} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \delta_0 t\, \phi(\omega, \theta) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$$

The indirect welfare effect described by the IW_0 term is an additional sufficient statistics one would need in order to calculate optimal marginal tax rates; it reflects how transfers would vary with a change in marginal tax rates at different incomes in the distribution.

Let us note that this term affects the welfare part of the above formula. Transfers adjusting to a

[39] Whereas it has no impact on the net earnings of individuals with productivity less than ω_0.

change in tax rates capture the indirect welfare effects of a change in taxes.

First, this term may be rewritten as:

$$IW_0 = \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 < \underline{\omega}^\theta} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \delta_0 t \phi(\omega, \theta) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta} + \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \delta_0 t \phi(\omega, \theta) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$$

This shows how the welfare term is affected, depending on y_0 being part of networks where transfers occur (second term) or not (first term).

Let us partition society in three different groups for each y_0.
Define $\mathcal{C}_0$ to be the set of groups in which individuals of ability ω_0 are present (according to the government). Then, $\overline{\mathcal{C}}_0$ is the set of groups in which they are not because their income is lower than all incomes in those groups. Finally, $\underline{\mathcal{C}}_0$ is the set of groups in which they are not because their income is higher than all incomes in those groups.

Then, quite naturally, an increase of taxes on incomes larger or equal to y_0 does not affect groups in the last set $\underline{\mathcal{C}}_0$. It does, however, affect groups in both $\mathcal{C}_0$ and $\overline{\mathcal{C}}_0$. The effects will be of different natures.[40]

Multiplicative effects. In groups belonging to $\overline{\mathcal{C}}_0$, all agents experience a small increase in tax liability τdy_0, which directly weighs on the lower-skilled individuals of these groups more than it does on the higher-skilled ones. Thus, from Lemma 1, we know that the higher skilled agents of these groups will compensate the lower skilled ones by increasing their transfers, thus reducing the welfare cost associated to them. So that even though consumption levels decrease for all agents, they decrease less than one-for-one for beneficiaries, which *dampens* the welfare effect on them. In that case, as the government typically associates more weight to lower skilled individuals, the welfare cost associated to individuals in groups belonging to $\overline{\mathcal{C}}_0$ is lower than it would have been without transfers. This effect is illustrated in Panel (b) of Figure 4.

Additive and multiplicative effects. In groups to which individuals of type ω_0 belong (groups in $\mathcal{C}_0$), there are two kinds of effects that are at play: an additive (spillover) and a multiplicative (dampening) one. Their combined effect on the final welfare cost is ambiguous.

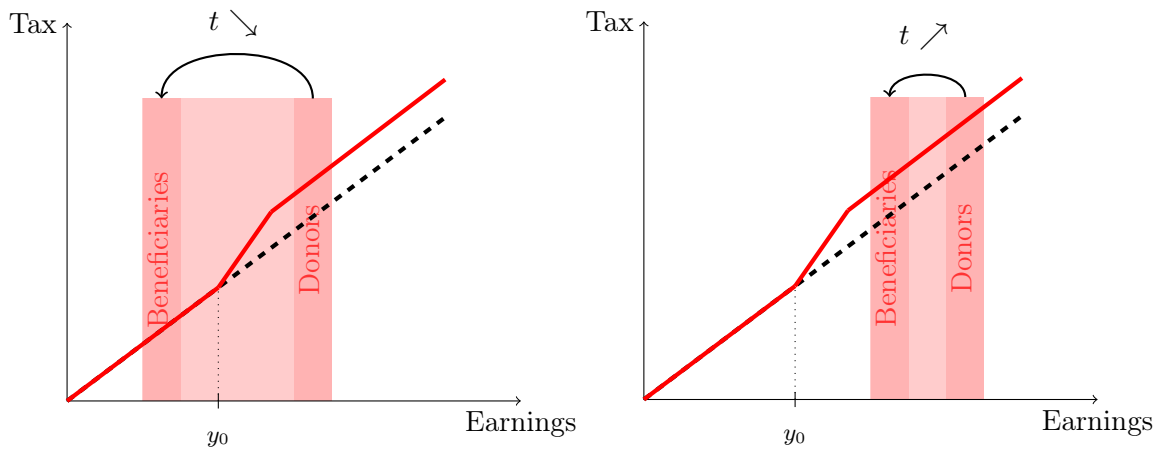
If the group is such that individuals directly affected by the tax increase (i.e., all individuals with skills higher or equal to ω_0) are neutral (neither donor nor beneficiaries) or donors, then the indirect welfare will consist in both additive and multiplicative effects. If individuals with skill ω_0 are neutral, then the net income of donors in their group is lower, so that the transfers they make decrease. Hence, beneficiaries in these groups are also affected even though their skill level is lower than ω_0: this adds an extra term to the welfare cost, a *spillover* effect. This effect is illustrated in Panel (a) of Figure 4. The same effect arises if the individuals of skill ω_0 are donors. Thus, through these extra terms, the welfare cost of increasing the tax liability of donors and neutral individuals is higher than it would have been without transfers: the additive effect is a spillover effect. However, there is also a dampening effect through the multiplicative term on the donors: indeed, for these individuals, since their transfers

[40] The combination of the direct and indirect welfare effects in mathematical terms can be found in Appendix B.

decrease, consumption varies less than one-for-one, so that the welfare cost that weighs on them is lower that it would have been without transfers.

In groups where individuals with skill ω_0 are beneficiaries, then again, both effects are at play. On the one hand, if they are not the lowest-skilled beneficiary in their group then the same kind of spillover (additive) effect occurs: the increase in tax liability of the donors in their group also affects the beneficiaries with skills lower than theirs (lower than ω_0). On the other hand, they might receive a similar compensation to that described for individuals in groups of type $\overline{\mathcal{C}}_0$: that is, their tax increase may be partly compensated by transfers made by the donors in their group. Moreover, this also translates into a multiplicative effect on donors.

Thus, the term IW_0 encompasses two types of indirect welfare effects: a multiplicative effect that dampens or amplifies the welfare cost associated to a uniform increase in tax liability (i.e., by transfers from donors to beneficiaries, whose final consumption levels are reduced more than a one-for-one factor for donors and less for beneficiaries), and a spillover effect that adds to the welfare cost by impacting individuals who would not have been impacted otherwise (i.e., beneficiaries whose tax liability is not increased but whose final consumption decreases as a result of decreased transfers from the donors in their group). The first kind of indirect effect – the multiplicative one – might lower or increase the welfare cost component depending on the relative welfare weights of donors and beneficiaries, as well as on their density: it may imply higher or lower marginal tax rates, but generally implies higher rates. The second kind of indirect effect – the additive one – unambiguously increases the welfare cost and implies lower marginal tax rates.



(a) Spillover effect
$\mathcal{C}_0$-type group

(b) Dampening effect
$\overline{\mathcal{C}}_0$-type group

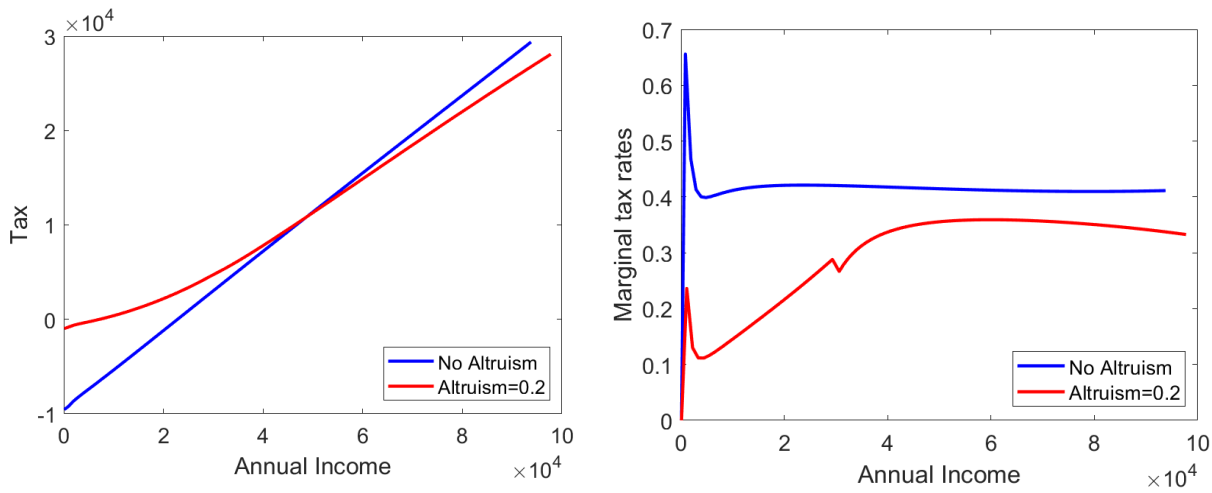
Figure 4: Indirect welfare effect

This figure illustrates the new effect induced by private transfers: the indirect welfare effect. This effect may go in two opposite directions depending on the position of groups with respect to income y_0. Panel (a) illustrates the effect of a slight increase in the tax rate if y_0 is the income of a Neutral individual in a group. This decreases available income for Donors and thus decreases the transfers they make. Panel (b) illustrates the effect of a slight increase in the tax rate at y_0 if all incomes in the group are above y_0. Then, for all CARA and DARA utility functions, this decreases the transfers made.

Example 2. (Non-linear income tax) *In both cases, B, the government's exogenous expenditure requirement is equal to 4000. Again we compare the tax schedule set by a "naive" Social Planner who would believe that there is no altruism with that set with a Social Planner who is aware of the level of altruism in society. Here, the Social Planner is (unweighted) utilitarian. Here, we calibrate the distribution of incomes (hence skill levels) on the United States in 2016 usig PSID data.*

Suppose we are in a Society of type A. That is, we only have one type of group (all groups are representative of the society as a whole). In the presence of CRRA utility functions and isoelactic costs of effort, the increase in consumption due to an infinitesimal increase in all disposable incomes from y_0 onwards for the beneficiaries ($\delta_0 Q_m$) is always greater than one (see Appendix C). Thus, the multiplicative effect is dampening on beneficiaries. However, the increase in consumption for donors ($\delta_0 Q_M$) may be lower or greater than one, so that the direction of its total effect is not clear.

In the following example, all groups follow the same log-normal distribution with a Pareto tail. Suppose they all have altruism level $\alpha = 0.2$. Then, marginal tax rates and the tax schedule are affected as follows:



(a) Tax schedule

(b) Marginal tax rates

Figure 5: Society A tax schedule

In Society A, all the groups have the same characteristics and span the whole society. Even though altruism is relatively low ($\alpha = 0.2$), marginal tax rates are much lower than in a non-altruistic setting: the spillover effect dominates. This in turn implies a much lower lump-sum transfer by the government, which may count on the higher-skilled individuals to redistribute to the lower-skilled individuals. This also enables the government to raise the required revenue.

In that case, the government raises revenue through two means: through the increase in incomes of all individuals (due to the lower efficiency cost of tax rates that are all lowered), and through lower lump-sum redistribution. It may rely on the donors in the groups to redistribute to the lower-skilled agents. As in Saez (2001), marginal tax rates are still U-shaped, but are considerably decreased on much of the population due to the spillover effects. Not that the sharp change in marginal tax rates occurs between neutral individuals' incomes and donors' incomes. The following figure shows a shift

in earnings to the right: lower marginal tax rates on the whole population imply a lower distortion on productive effort decisions and hence an increase in all earnings.

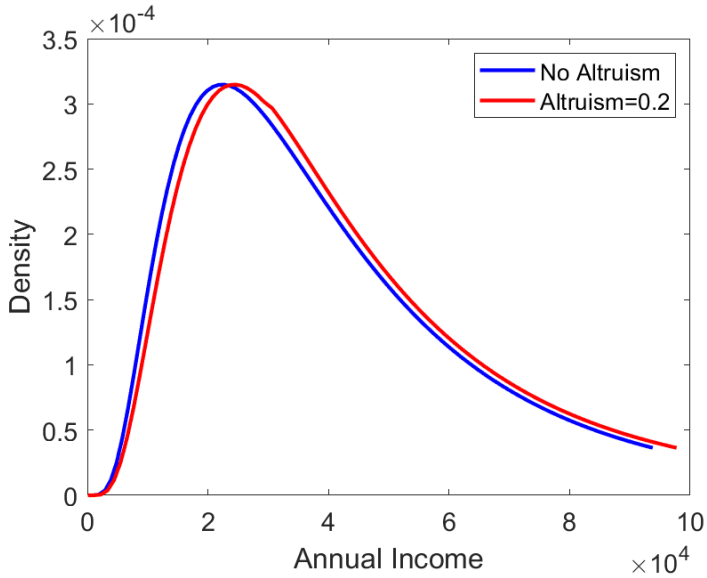
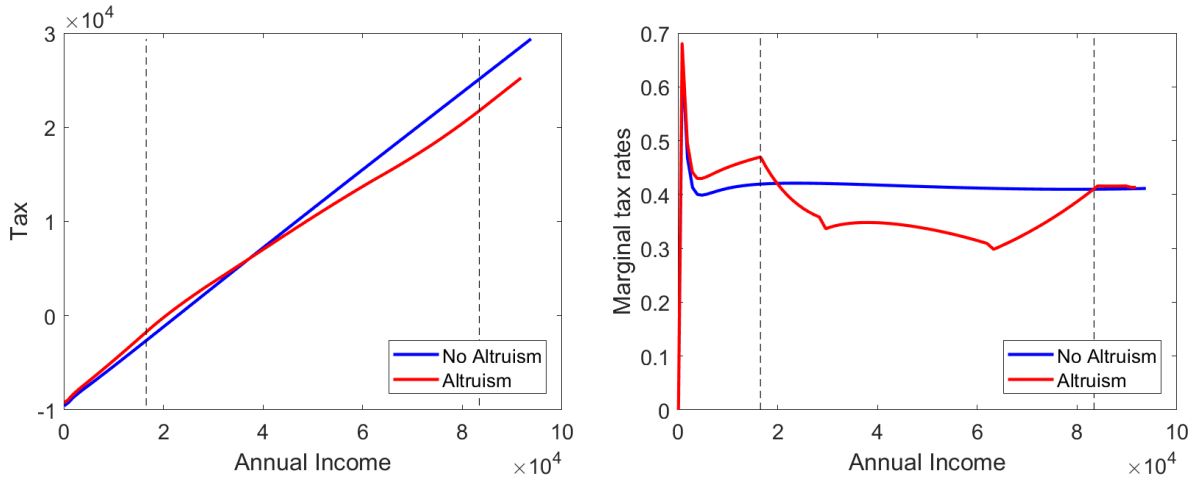


Figure 6: Distribution of earnings in Society A

In Society A, marginal tax rates are much lower on all incomes and hence imply a lower distortion on productive effort.

In Society B, now, the distribution of skills is the same at the society level as a whole. There are three types of groups: lower skilled groups that comprise 19 percent of the population, medium skilled groups that comprise the next 70 percent of the population, and finally the highest skilled groups that comprise the last 11 percent of the population. In that case, as can be seen in Figure 7, lump-sum redistribution is hardly affected, but marginal tax rates are very different: they may be increased on the lowest-skilled group in order to reach a high enough tax liability for higher skilled individuals while at the same time lowering the rates on the middle income group in order to encourage transfers at the group-level while raising more revenue. For higher skilled individuals, marginal tax rates are hardly affected. Indeed, not enough redistribution takes place in that group for the government to encourage it. For the first type of groups, the dampening effect is at play: it increases tax rates. For the second type of group, both dampening (as seen for the tax rates of a part of beneficiaries) and spillover effects are at play (as seen on the rest of the group): on these groups, tax rates are generally lower. Again, the sharp changes in marginal tax rates in the middle income group occurs when changing from beneficiaries' incomes to neutral individuals' and then from the latter's to donors'.



(a) Tax schedule

(b) Marginal tax rates

Figure 7: Society B tax schedule

Society B is composed of three types of groups. The lower-skilled and higher-skilled groups are homogeneous and hence operate little redistribution. The middle-income group spans a large set of skill levels and hence operates redistribution absent government intervention. This example enables to highlight both the spillover and the dampening effects. The higher marginal tax rates on the poorer individuals are due to the dampening effect. The lower marginal tax rate on middle income individuals due to spillover effects. Finally, the marginal tax rates on richer individuals are almost unchanged. The lump-sum transfer is now also hardly unchanged: the government has to take care of redistributing to the very poor.

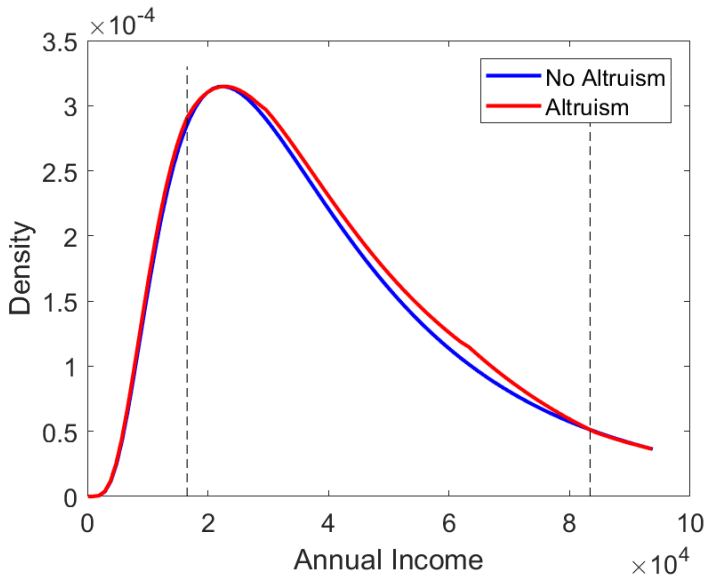


Figure 8: Distribution of earnings in Society B

Here, the distortion on productive effort decisions is only lower for middle-income individuals.

Earnings are now higher only for the middle income group.

5 Extensions

5.1 Taxation in developing countries

As discussed in the Introduction, private transfers are pervasive in developing countries. A large part of the Public Finance literature in developing countries is focused on the emergence of tax systems and State capacity in such settings.
This framework, however, might provide some insights into the *design* of tax systems in these contexts. I extend it in two dimensions to account for important features of developing countries.

5.1.1 Kinship taxation

Social norms are an important factor explaining the presence of private transfers. While altruism might also be thought of as a social norm, it is not, as we have seen in the above analysis, a distortive one. Indeed, in the absence of income effects, productive effort decisions are not affected by transfers. Moreover, if they were, higher productivity individuals would exert more effort while lower productivity individuals would exert less. Thus, the resulting aggregate income would be higher than if there were no altruism. This is not, however, the kind of distortion that is described in the literature (see Baland et al. (2011), Jakiela and Ozier (2015), Squires (2016)): part of these transfers are also found to discourage productive effort. These kinds of transfers are due to a kinship tax, that exists for social norm purposes.

In order to illustrate the new effects that are at play now, let us place ourselves in a society where there are two groups, say C_1 and C_2. Within each group, individuals have altruistic ties towards their peers. Between both groups, there is a kinship tax. Thus, individuals observe both skill levels and earning levels within their group, and only earnings in the other group. Suppose both groups have the same variance in skill levels, but C_1 has a higher mean than C_2. Drawing from the empirical findings in Squires (2016), I model the kinship t_K tax as follows:

$$t^K(y) = \begin{cases} \tau^K \times (y - y^K) & \text{if } y \geq y^K \\ 0 & \text{otherwise} \end{cases}$$

Where τ^K and y^K are not the result of some maximization program, but are set by those imposing the tax. Tax proceeds are then split equally between these individuals. We suppose that ω^K the productivity level of individuals with earnings y^K when there is no tax is such that $\underline{\omega}^1 < \omega^K < \overline{\omega}^1$, and $\omega^K > \overline{\omega}^2$. Thus, the kinship tax is imposed on some individuals of group C_1 and redistributed lump-sum in group C_2. Now, when there is no government intervention, individuals with productivity level ω^K choose the same productive effort y^K as they would without the kinship tax. Individuals with incomes between ω^K and $\omega^K + \delta^K$ for some δ_K bunch at y^K, and individuals with higher productivity levels choose their productive effort according to the first order condition

$$\frac{1}{\omega} k' \left(\frac{y}{\omega} \right) = 1 - \tau^K$$

Let us now comment on the *laissez-faire* situation.

Incomes are still non decreasing in productivity levels within each group, so that transfers still have the same form: they flow from higher skilled to lower skilled individuals, and directly from donor to receiver.

Now, however, higher skilled individuals in group C_1 earn less. Thus, there is the possibility that they do not make any transfers in their group because of this kinship tax. At the same time, however, the kinship tax enables to make the poorer group, C_2, richer in terms of final consumption level, while at the same time potentially discouraging the individuals in it to make altruistic transfers to their poorer members and while now potentially making them richer than the poorer individuals in the taxed group C_1. While the kinship tax induces an efficiency loss, it is also redistributive.

In this context it is interesting to consider what a social Planner would want to do. The tax introduced by the State will erode the tax base for the kinship tax – thus reducing inefficiencies caused by this tax, while at the same time introducing a new one. Crucially, the interaction between formal and informal redistribution will depend on whether τ^K, and especially y^K are endogenous to the groups' characteristics or not – for example, depend on relative income levels. For instance, we may suppose that $y^K = \gamma^K \times \big(y(\overline{\omega}^2) - T(y(\overline{\omega}^2)\big)$, where γ^K is some exogenously set positive integer. Thus, y^K is endogenous to the tax.

The Social Planner now wants to maximize the sum of the following two terms:

$$\int_{\underline{\omega}^1}^{\overline{\omega}^1} \Phi\left(v\left(y(\omega,\tau^K) - T(y(\omega,\tau^K)) - t^K[y(\omega,\tau^K) - T(y(\omega,\tau^K))] - t(\omega,\boldsymbol{\theta}_1,\tau^K) - k\left(\frac{y(\omega,\tau^K)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega$$

and $\displaystyle\int_{\underline{\omega}^2}^{\overline{\omega}^2} \Phi\left(v\left(y(\omega) - T(y(\omega)) - t(\omega,\boldsymbol{\theta}_2,L^K) + L^K - k\left(\frac{y(\omega)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega$

where $L^K = \displaystyle\int_{\underline{\omega}^1}^{\overline{\omega}^1} t^K(y(\omega,\tau^K) - T(y(\omega,\tau^K)))\, h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega$,

under the budget constraint $\displaystyle\int_{\underline{\omega}^1}^{\overline{\omega}^1} T(y(\omega,\tau^K))\, h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega + \int_{\underline{\omega}^2}^{\overline{\omega}^2} T(y(\omega))\, h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega \geq 2B.$

And subject to individual behaviors. The structure of information is such that groups know each other, but the government does not. It does know, however, τ^K and either y^K if it is exogenously set, or else the shape of y^K (for example γ^K in the above example). The government may rely on the information that individuals have about each other to design a tax system such that

In order to determine the optimal income tax rates, let us use the perturbation method.

Start from an initial tax T and define ω'_K the new productivity level such that $y(\omega'_K,\tau^K) - T(y(\omega'_K,\tau^K) = y^K$. Then, write $y' = y(\omega'_K,\tau^K)$.
Define ω^1_m, ω^1_M, ω^2_m, ω^2_M as usual.

Again, we may write $\frac{dt}{d\tau} = dy_0\delta_0 t$, but this will now be different. Indeed, it now includes the indirect effect on L^K. For the extracting group, C_1, L^K decreases by an infinitesimal amount for all

individuals, so that again we might get the dampening or amplifying effect. For the extracted group C_2, the kinship tax burden is reduced, thus encouraging more transfers to lower skilled individuals in the group. Thus, even though we now have an extra term through $\frac{dL^K}{d\tau}$, $\delta_0 t$ may have a smaller effect: effects go in opposite direction.

Proposition 4 *The marginal tax rates are of the following form:*

$$\frac{T'(y_0)}{1-T'(y_0)} = \left(\int_{y_0}^{\infty}(1-\phi(y))\frac{f_y(y)}{1-F_y(y_0)}\,\mathrm{d}y + \frac{IW_0}{1-F_y(y_0)}\right) \times \frac{1-F_y(y_0)}{y_0 f_y^*(y_0)} \times \frac{\mathbb{1}\{y_0 \geq y(\omega'^K)\}(1-\tau^K)}{e(y_0)}$$

IW_0 depends on whether individuals are in the extracting group, are in the extracted group but do not pay the kinship tax, or are in the extracted group and do pay the kinship tax.

If the individual of skill ω_0 is in group C_2, the extracting group, then

$$IW_0 = \int_{\underline{y}^1}^{\overline{y}^1} \delta_0 t\,\phi(y) f_1(y)\,\mathrm{d}y + \int_{\underline{y}^2}^{\overline{y}^2} \delta_0 t\,\phi(y) f_2(y)\,\mathrm{d}y + \tau^K(1-\delta_0 y^K)\int_{y(\omega'^K)}^{\overline{y}^1} \phi(y)\,f_1(y)\,\mathrm{d}y$$

$$-\tau^K(1-\delta_0 y^K)\int_{y(\omega'^K)}^{\overline{y}^1} f_1(y)\,\mathrm{d}y \int_{\underline{y}^2}^{\overline{y}^2} \phi(y)\,f_2(y)\,\mathrm{d}y.$$

If the individual of skill ω_0 is in group C_1, the extracted group, but does not pay the kinship tax, then

$$IW_0 = \int_{\underline{y}^1}^{\overline{y}^1} \delta_0 t\,\phi(y) f_1(y)\,\mathrm{d}y + \int_{\underline{y}^2}^{\overline{y}^2} \delta_0 t\,\phi(y) f_2(y)\,\mathrm{d}y + \tau^K(1-\delta_0 y^K)\int_{y(\omega'^K)}^{\overline{y}^1} \phi(y)\,f_1(y)\,\mathrm{d}y$$

$$-\tau^K(1-\delta_0 y^K)\int_{y(\omega'^K)}^{\overline{y}^1} f_1(y)\,\mathrm{d}y \int_{\underline{y}^2}^{\overline{y}^2} \phi(y)\,f_2(y)\,\mathrm{d}y.$$

If the individual of skill ω_0 is in group C_1, the extracted group, and pays the kinship tax, then

$$IW_0 = \int_{\underline{y}^1}^{\overline{y}^1} \delta_0 t\,\phi(y) f_1(y)\,\mathrm{d}y + \int_{\underline{y}^2}^{\overline{y}^2} \delta_0 t\,\phi(y) f_2(y)\,\mathrm{d}y + \tau^K(1-\delta_0 y^K)\int_{y_0}^{\overline{y}^1} \phi(y)\,f_1(y)\,\mathrm{d}y$$

$$-\tau^K(1-\delta_0 y^K)\left(e(y_0)y_0 f_1^*(y_0) + \int_{y_0}^{\overline{y}^1} f_1(y)\,\mathrm{d}y\right)\int_{\underline{y}^2}^{\overline{y}^2} \phi(y)\,f_2(y)\,\mathrm{d}y.$$

There is now a trade-off between eliminating the kinship tax, which implies reducing altruistic transfers and having the distortion on higher incomes only originating from the formal tax (instead of imposing a double tax on individuals who are already subject to the kinship tax). It is now not necessarily more efficient to decrease marginal tax rates on the group which imposes the kinship tax, as it encourages this informal tax and thus increases the double distortion on higher incomes. The higher τ^K is, the more the government wants to crowd-out the kinship tax.

Now, taxes on incomes in group C_1 also have an effect on transfers in group C_2. Because the lump sum they receive from individuals in C_1 decreases, they increase the transfers they make to the poorest in their group. So decreasing the kinship tax *crowds in* altruistic transfers in the extracting group, and has an ambiguous effect in the extracted group. On the one hand it increases the formal tax paid by donors, but on the other hand it also decreases the kinship tax they pay: hence, they might increase or decrease the transfers they make. Finally, the double distortion exerted on those who are extracted has a repercussion on L^K.

5.2 Family taxation

Here we relax the assumption that the government cannot observe groups. The government can observe parents and children, knows that children have no productive ability, but does not know the skill level of parents. In this case, it can be shown that the number of children in a household represents a tag for the government on the consumption level of the parents. At the same time, if the children enter the social welfare function of the social planner, the social planner has an interest in encouraging private redistribution from parents to children. This might explain why in certain countries, such as France, not only do families with children receive benefits (lump-sum transfers), but they also see their tax rates decreased.

6 Concluding remarks

This paper develops a conceptual framework to study optimal taxation in the presence of altruism and thus private transfers between individuals. First, I establish the structure of incomes and transfers when there is no government intervention. Second, I characterize the optimal linear and non-linear income taxes and show that depending on the distribution of the homogeneity of groups, because of partial crowding out transfers, the optimal tax rates should take into account the reactions transfers to the tax rate. Indeed, the social planner might find it advantageous to encourage transfers at a private level before operating redistribution at the society level: by taking into consideration the partial crowding out of transfers, the government may take advantage of the fact that there is already some redistribution occurring at a private level – a redistribution that does not distort labor supply incentives – and thus decrease the marginal tax rate used to redistribute at the society level – thus also reducing the amount of distortion created by the tax. This framework can then be extended to a number of settings, such as family taxation and taxation in developing countries.

Appendix A: Transfer patterns in a selection of countries

In this section, I use data from the Living Standard Measurement Surveys (LSMS) from the World Bank, that often include a section on private transfers between households. Even though these surveys are country-specific and do not as such allow for a comparison between countries (see Cox et al. (2006)), they allow me to provide some descriptive evidence on the extent, patterns and reasons for private transfers for six different countries for which this data was available in the last ten years: Burkina Faso, Ghana, Mali, Niger, Tajikistan and Uganda. I always consider the latest year available, which I specify at the beginning of each paragraph.

In 2017, these six countries are classified as low income countries by the World Bank (i.e., their Gross National Income per capita is below 996 current US $) and their government expenditure as a percentage of GDP is between 8% (for Ghana) and 18% (for Tajikistan, closely followed by Mali with 16.9% and Niger with 15.8%). Of course this section only aims to provide some descriptive evidence on transfers in these countries, and I shall only add some potential hypotheses for interpretation.

Burkina Faso – 2013. The LSMS provide very detailed data on private transfers between households in Burkina Faso. However, despite asking how often transfers were made or received in the past year, they do not say if the transfers are made on a regular basis or due to a temporary shock. Indeed a transfer made or received once a year may be yearly or one-shot in order to cover some unexpected expense. One benefit of this database is that it is specified at the very beginning of the section that it concerns transfers that are not made in exchange for anything. Thus, they enable us to eliminate the reciprocity motive (in the short term at least), but not consumption-smoothing transfers (i.e. due to a temporary shock).

Private transfers are widespread in Burkina Faso: 34% of households report having received at least a transfer in the past year, and 39% having made one. 66% of households indicate having received transfers more than 20 times in a year, and 62% having made more than 20. The main reason for transfers is family support, with health and education coming (quite far) behind – which might be an indication for regular transfers (special events such as funerals or weddings only represent 4% of transfers). More than half of the transfers are either given in person or carried by someone else – thus not directly observable by the State. More than two thirds of transfers are made by households living in Burkina Faso, and 96% of recipients live in Burkina. Moreover, they live in the same locality in 40% of cases, or in a rural area one fourth of the time. More than 60% of transfers are monetary. 33% of donors and 44% of recipients are not in the nuclear family, thus presenting evidence for widespread networks of individual links. Finally, 38% of recipients are students or unemployed – evidence perhaps for transfers that are redistributive, in the short-run at least. Below are some graphs presenting some of this evidence.

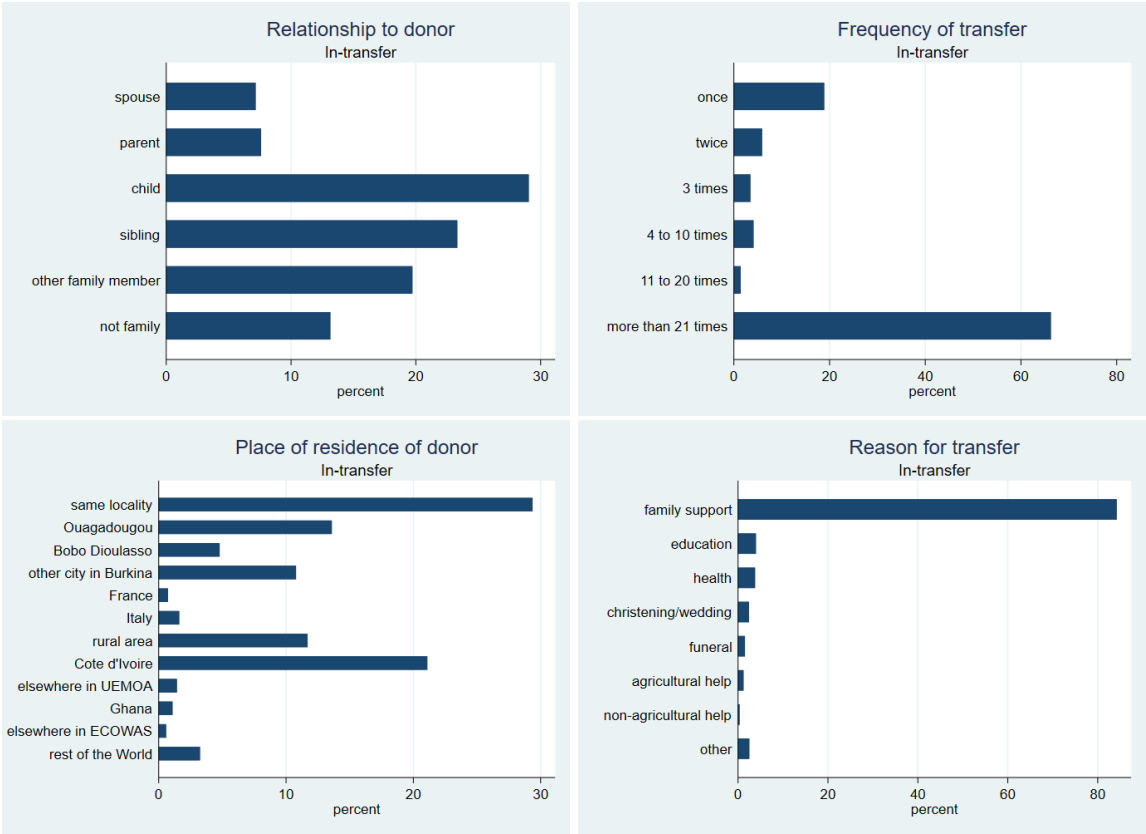


Figure 10: Characteristics of transfers received: Benin

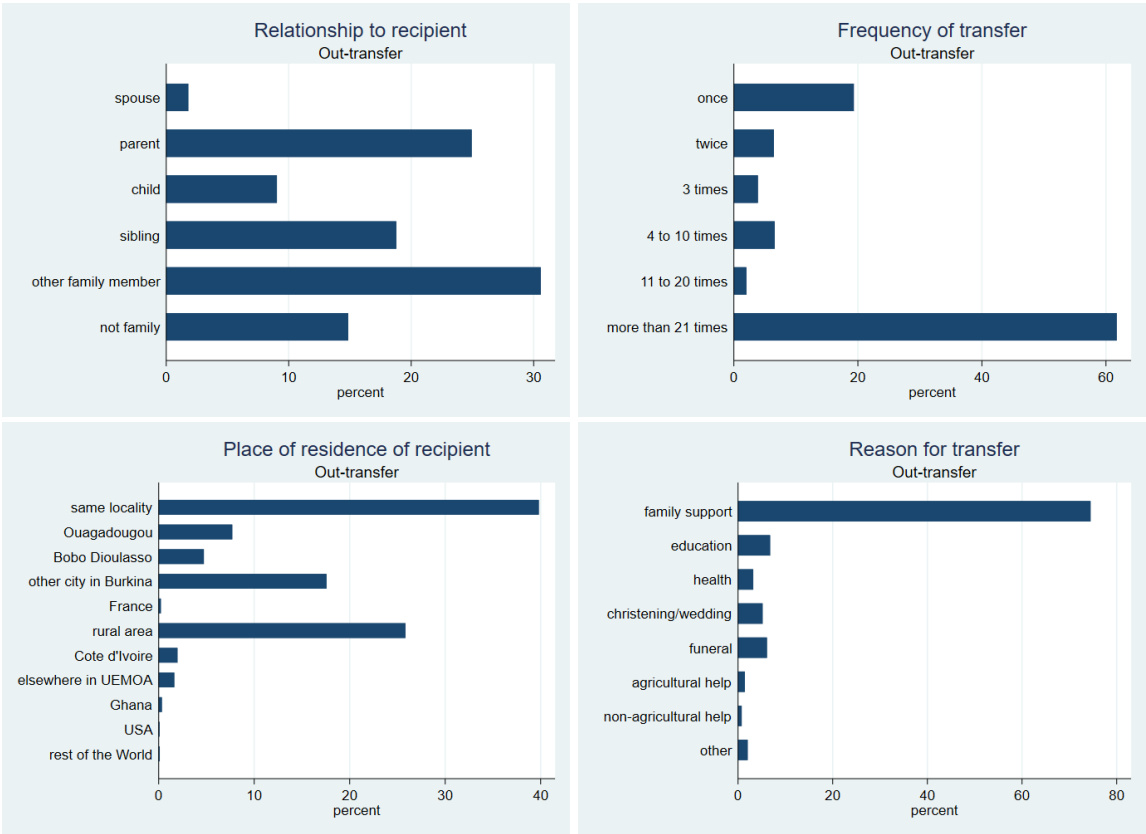


Figure 12: Characteristics of transfers made: Benin

Ghana – 2009. While the questionnaire for Ghana does not explicitly ask whether transfers take place in exchange for services, they do ask how regular the transfers are. Half of the transfers are regular (i.e., on a yearly, monthly, weekly or even more regular basis). When considering amounts, more than two-thirds of the total stock of transfers is done on a regular basis. 83% of transfers are received from a relative and 89% made to a relative. 55% of transfers received are used for daily consumption, while 16% are for health purposes, and 7% are for education support. For transfers made, these numbers are respectively 56, 12 and 19. The three principal reasons for transfers are the same as for Benin. Again, funerals and other ceremonies represent less than 7% of the reasons for which transfers occur. Transfers mostly flow from urban areas in Ghana (in 70% of cases – among which 24% from the capital, Accra), even though 12% of the time they originate from the same town or village. Only 17% originate from abroad. Finally, adding again to the evidence that transfers are not directly observable by the government is the fact that more than three fourths of transfers are made hand-to-hand.

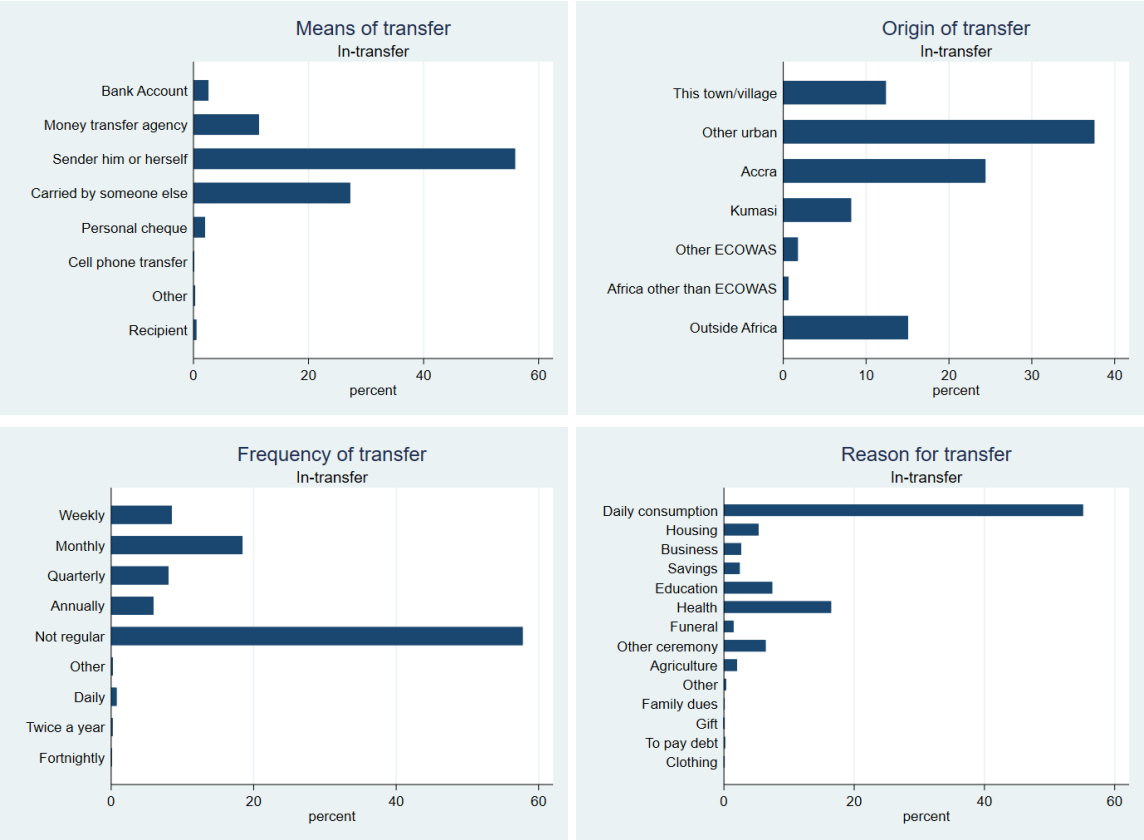


Figure 14: Characteristics of transfers received: Ghana

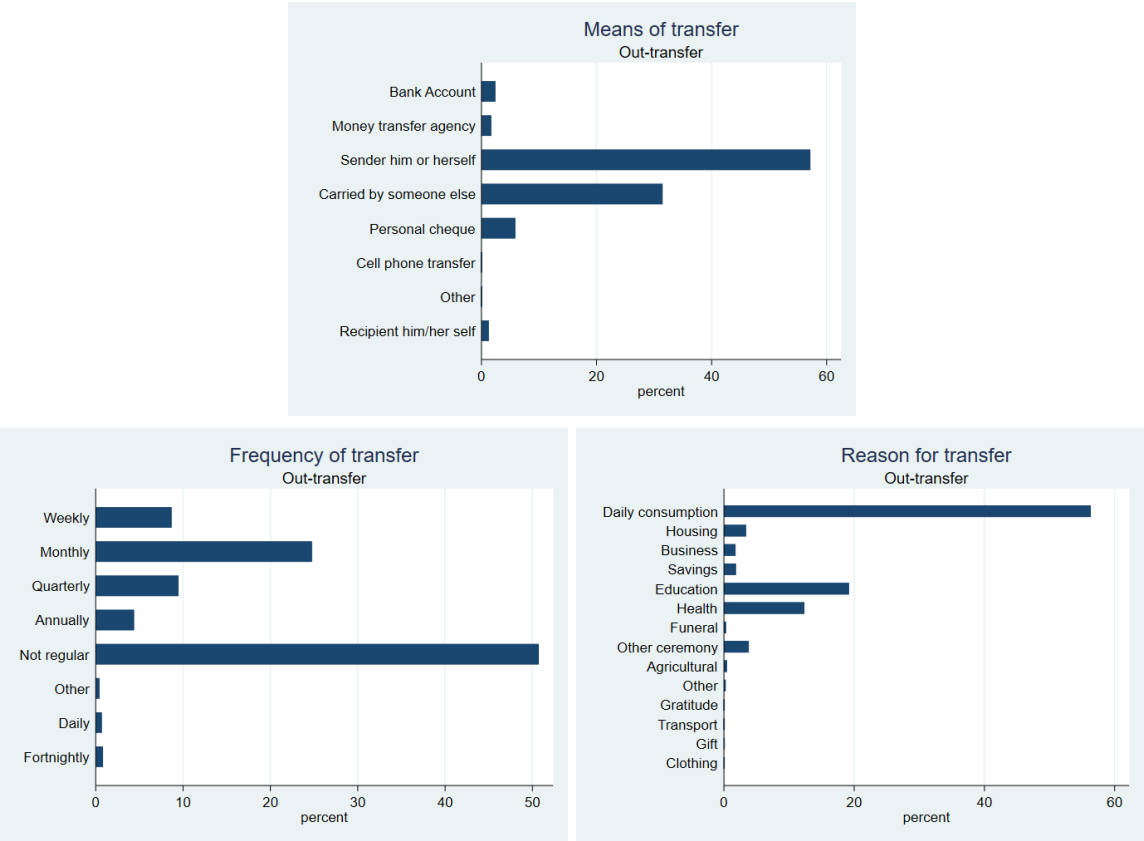


Figure 16: Characteristics of transfers made: Ghana

Mali – 2014. The questionnaire for Mali also includes a question on the frequency of transfers. Again, an important share of transfers occur on a regular basis: 45% (respectively 38%) of transfers received (respectively made) are yearly, half-yearly, quarterly or monthly. Transfers mostly occur from children to parents but also between siblings – thus calling into question a model where a family head would be altruistic towards their family members, with no direct altruistic links between each other (if these transfers are made on a voluntary basis). Transfers to non-family members are rare, but transfers to or from non-nuclear-family members represent around 15% of transfers made or received. In around two-thirds of the cases, the donor lives in Mali. One fourth of transfers originate from Bamako (the capital), and 9% from the same locality. Transfers are generally made within Mali (in 91% of cases), and in 17% of cases to the same locality. Unless all transfers occurring outside of the household's locality are non-altruistically motivated,[41] these figures (along with those for Ghana) perhaps provide evidence that it may be difficult for a government to know group-belonging. Pointing in this direction again, 35% of donors and 65% of recipients never lived in the household they give to (resp. receive from). For the most part, recipients live in rural areas (65%), but donors are almost evenly distributed between rural and urban areas (resp. 47 and 53%). The principal reason for transfers is still ongoing support, but here, transfers related to education and health take on less or as much importance as transfers related to special events and agricultural help.

[41] Which would be contrary to the idea that it is easier to exert pressure on individuals who are closer geographically

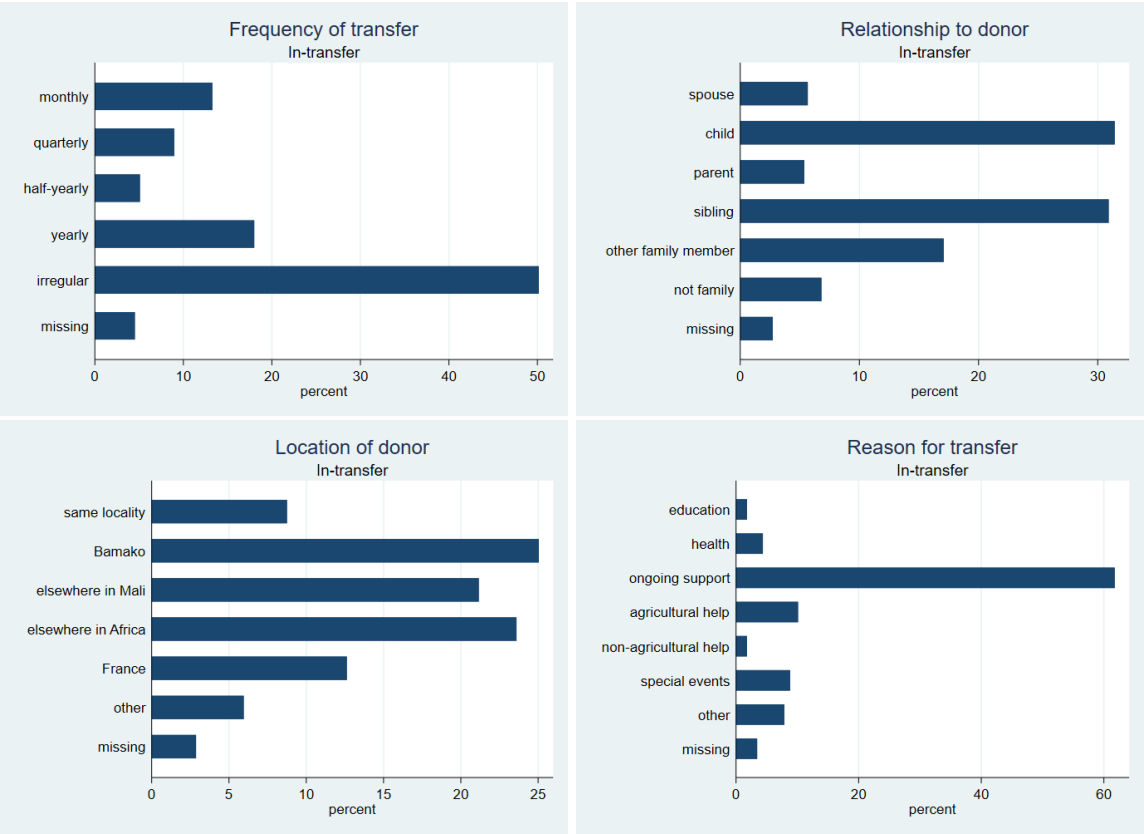


Figure 18: Characteristics of transfers received: Mali

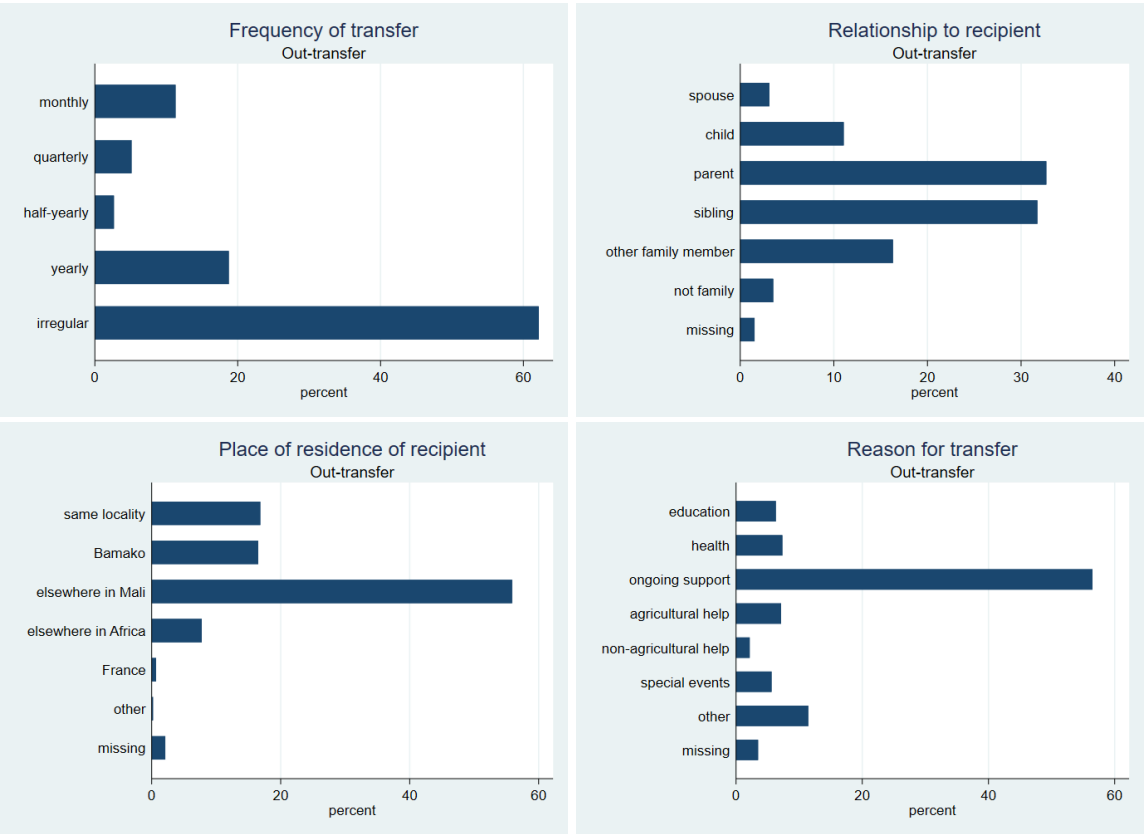


Figure 20: Characteristics of transfers made: Mali

Niger – 2014. Here, family support and education might have been confounded, as one is the main reason for transfers made, while the other is the main reason for transfers received. A majority of donors have never lived in the household (55%). More than half of the transfers are monetary. Transfers flow in networks that seem quite wide: from children to parents, between siblings, but also between extended family members and between individuals who are not part of the same family (11% of donors are not family members, and 17% of receivers are not family members). Over half of donors are based in Niger, but recipients of transfers from Nigerien households are primarily from the same locality (in 55% of cases); and hardly none live outside of Niger (only 5%).

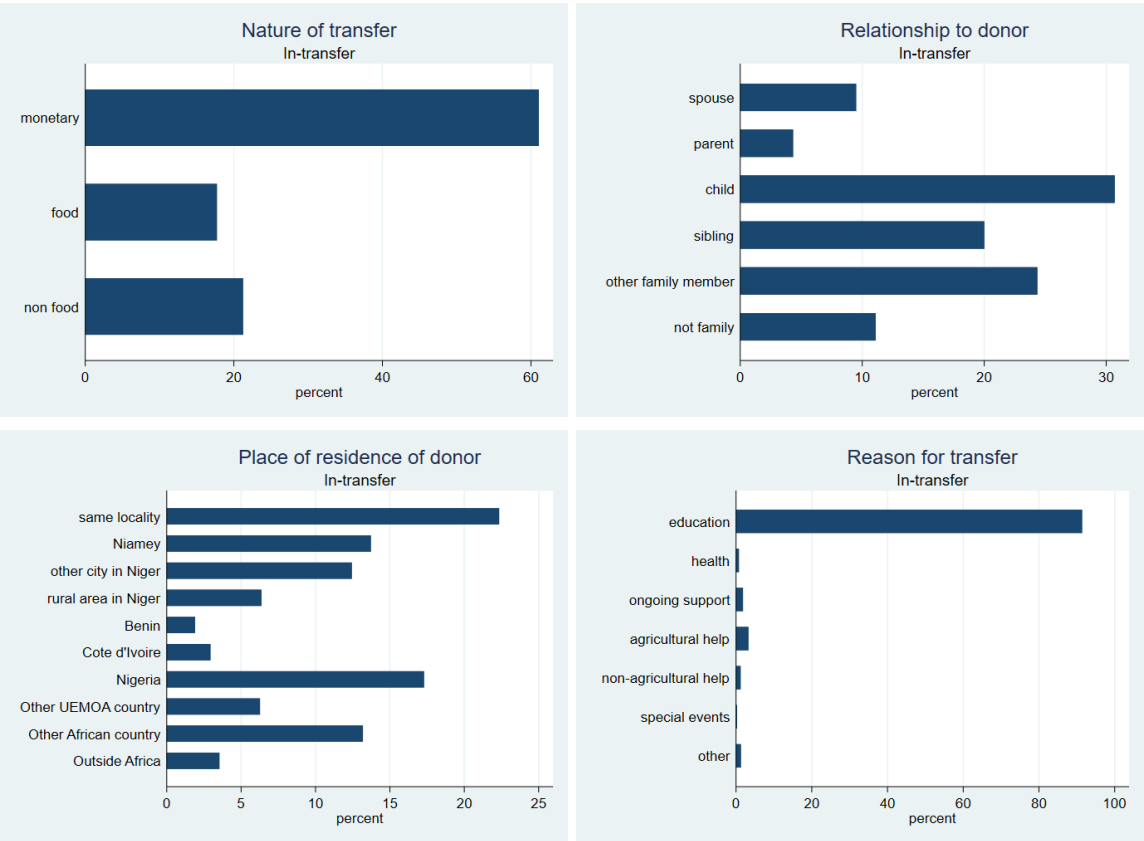
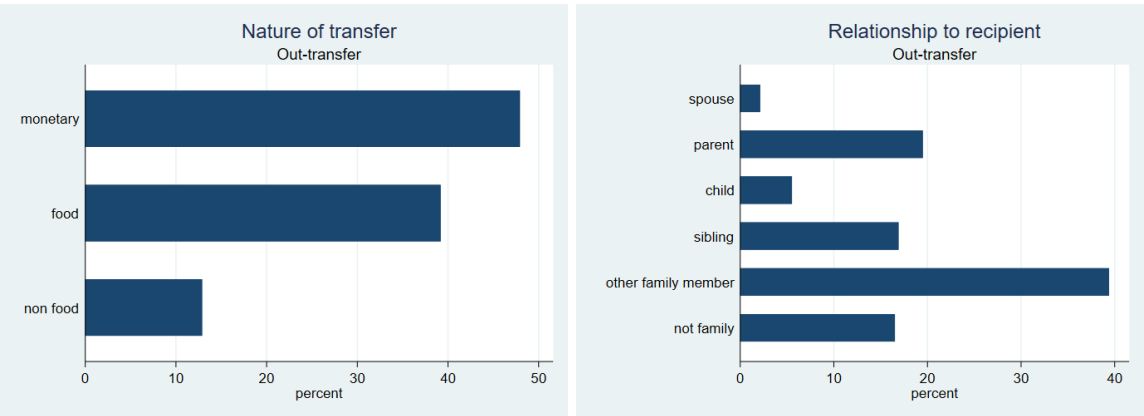


Figure 22: Characteristics of transfers received: Niger



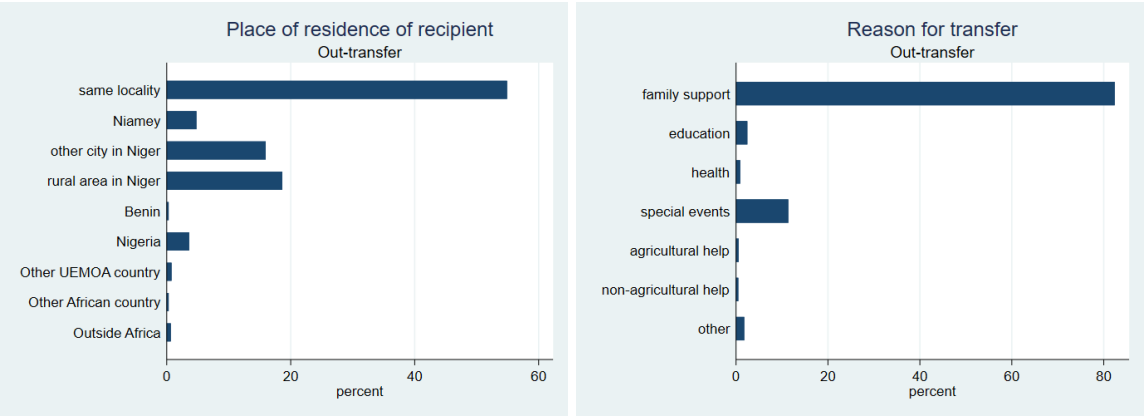


Figure 24: Characteristics of transfers made: Niger

Tajikistan –2009. 68% of donor are based in Tajikistan itself (with the remaining being based in Russia, mostly). Again, most transfers flow from children to parents or between siblings – even though 8% of transfers originate from non-family members. Transfers are mostly received for daily consumption (71% of transfers received) – perhaps indicating regular transfers, but are given out in 38% of cases for weddings, funerals or medical expenses. Still, 39% of transfers made remain for daily consumption purposes.

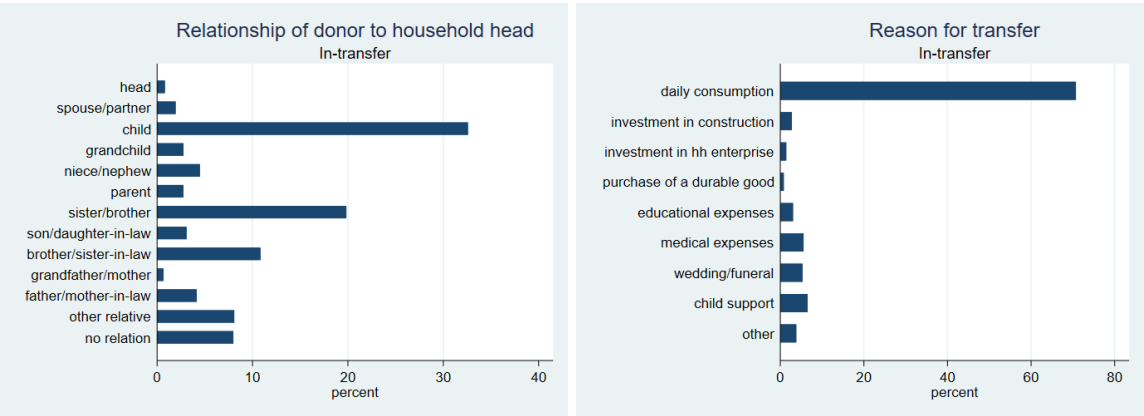


Figure 25: Characteristics of transfers received: Tajikistan

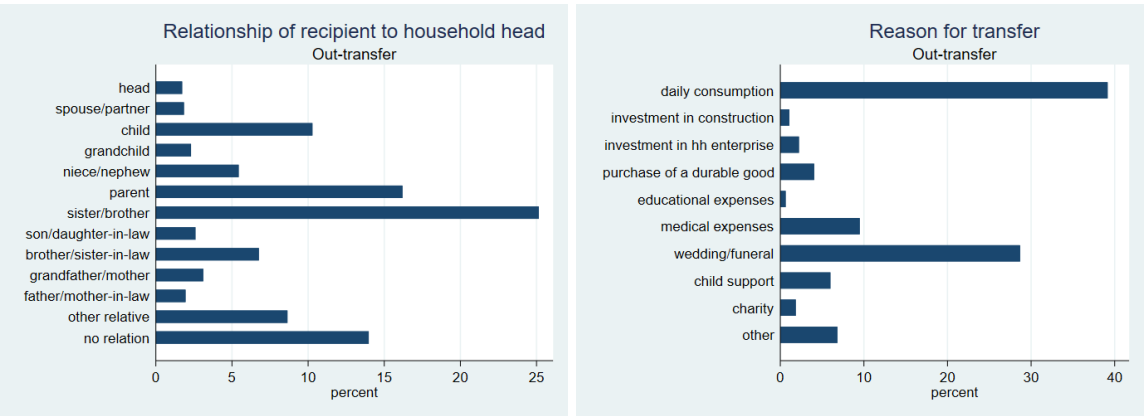
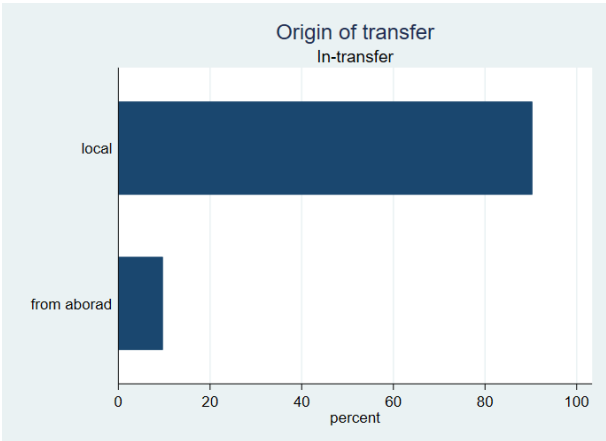
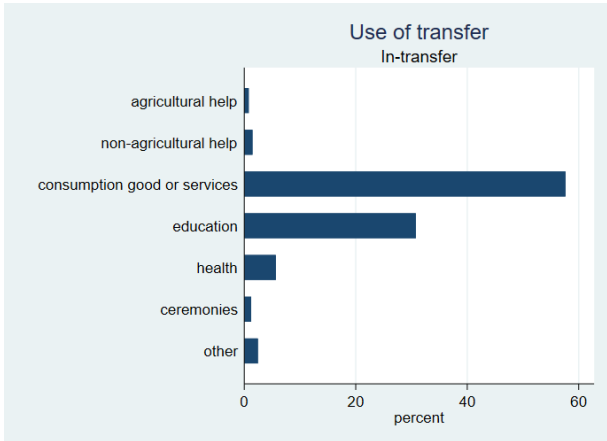


Figure 26: Characteristics of transfers made: Tajikistan

Uganda – 2013 90% of donors are based in Uganda. Moreover, 58% of transfers are received for daily consumption, and 31% to pay for education expenses – again, a potential indication that transfers flow on a regular basis.





Let us note in a general conclusion to this section that in all these examples, transfers do not only flow within the same village or locality. A large share of transfers originate from or are addressed to people who do not live in the same place. Thus, it might be hard for a government to know groups.

Appendix B: Simulations

This section is devoted to understanding how the indirect welfare effect can be rewritten in a more tractable way in other to run numerical simulations with different values of altruism α. Indeed, our term of interest – and for which we want a numerically applicable formula – is IW_0. At the end of the section we can also see how both the direct and indirect welfare effects add up and thus allow for a mathematical interpretation of the effects described in Section 4.2.

Numerically implementable formula. Let us start by defining ω_m^θ and ω_M^θ as the productivity levels in group of characteristics $\boldsymbol{\theta}$ to be such that, under the optimal tax schedule $T(.)$, individuals with productivity levels below ω_m^θ are receivers, and above ω_M^θ are donors.

For ease of notation, let us also introduce the following notation:

$$Y(\omega) = y(\omega) - T(y(\omega)) - k\left(\frac{y}{\omega}\right).$$

Now:

$$IW_0 dy_0 = -\int_{\boldsymbol{\theta}\ s.t.\ \omega_0 < \underline{\omega}^\theta} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \frac{\mathrm{d}t}{\mathrm{d}\tau} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta} - \int_{\boldsymbol{\theta}\ s.t.\ \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \frac{\mathrm{d}t}{\mathrm{d}\tau} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$$

Let us rewrite the two terms as follows:

First we know that in the first term where $\omega_0 < \underline{\omega}^\theta$, we have that $\omega_0 < \omega_m^\theta < \omega_M^\theta$.

Then, in the second term, ω_0's position with respect to ω_m^θ and ω_M^θ is important.
Let us recall that for receivers $t = Y - Q_m$ and for donors, $t = Y - Q_M$. Hence, with the perturbation described in the non-linear tax section, we have that:

- If $\omega_0 < \underline{\omega}^\theta$, then:
  - For receivers, $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -dy_0 - \dfrac{\mathrm{d}Q_m}{\mathrm{d}\tau}$.
  - For donors, $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -dy_0 - \dfrac{\mathrm{d}Q_M}{\mathrm{d}\tau}$.
- If $\omega_0 < \omega_m^\theta$, then:
  - For receivers with $\omega < \omega_0$ $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -\dfrac{\mathrm{d}Q_m}{\mathrm{d}\tau}$.
  - For receivers with $\omega \geq \omega_0$ $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -dy_0 - \dfrac{\mathrm{d}Q_m}{\mathrm{d}\tau}$.
  - For donors, $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -dy_0 - \dfrac{\mathrm{d}Q_M}{\mathrm{d}\tau}$.
- If $\omega_0 \in [\omega_m^\theta, \omega_M^\theta]$, then:
  - For receivers, $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -\dfrac{\mathrm{d}Q_m}{\mathrm{d}\tau}$.
  - For donors, $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -dy_0 - \dfrac{\mathrm{d}Q_M}{\mathrm{d}\tau}$.
- If $\omega_0 > \omega_M^\theta$, then:
  - For receivers, $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -\dfrac{\mathrm{d}Q_m}{\mathrm{d}\tau}$.
  - For donors with $\omega < \omega_0$, $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -\dfrac{\mathrm{d}Q_M}{\mathrm{d}\tau}$.
  - For donors with $\omega \geq \omega_0$, $\dfrac{\mathrm{d}t}{\mathrm{d}\tau} = -dy_0 - \dfrac{\mathrm{d}Q_M}{\mathrm{d}\tau}$.

For any group with characteristics $\boldsymbol{\theta}$, if transfers take place, then we know that:

$$\int_{\omega_M^\theta}^{\overline{\omega}^\theta} \left(Y(\omega) - Q_M^\theta\right) h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega = \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \left(Q_m^\theta - Y(\omega)\right) h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega.$$ [42]

We have the following four cases:

- If $\omega_0 < \underline{\omega}^\theta$, then the tax is increased by a small lump-sum amount on the whole group of type $\boldsymbol{\theta}$. Thus, taking the the derivative of our above equality with respect to τ implies that:

$$\frac{\mathrm{d}Q_m^\theta}{\mathrm{d}\tau} H(\omega_m^\theta|\theta) + \frac{\mathrm{d}Q_M^\theta}{\mathrm{d}\tau}(1 - H(\omega_M^\theta|\theta)) = -dy_0[H(\omega_m^\theta|\theta) + 1 - H(\omega_M^\theta|\theta)]$$

[42] Where Q_M, Q_m, and ω_m, ω_M are defined at the optimal tax schedule.

- If $\omega_0 < \omega_m^\theta$ then some receivers only perceive a change in transfers, but none in taxes, and:

$$\frac{\mathrm{d}Q_m^\theta}{\mathrm{d}\tau} H(\omega_m^\theta|\theta) + \frac{\mathrm{d}Q_M^\theta}{\mathrm{d}\tau}(1 - H(\omega_M^\theta|\theta)) = -dy_0[H(\omega_m^\theta|\theta) - H(\omega_0|\theta) + 1 - H(\omega_M^\theta|\theta)]$$

- If $\omega_0 \in [\omega_m^\theta, \omega_M^\theta]$, then, all donors have an increase in tax liability:

$$\frac{\mathrm{d}Q_m^\theta}{\mathrm{d}\tau} H(\omega_m^\theta|\theta) + \frac{\mathrm{d}Q_M^\theta}{\mathrm{d}\tau}(1 - H(\omega_M^\theta|\theta)) = -dy_0[1 - H(\omega_M^\theta|\theta)]$$

- If $\omega_0 > \omega_M^\theta$ then only some donors have an increase in tax liability:

$$\frac{\mathrm{d}Q_m^\theta}{\mathrm{d}\tau} H(\omega_m^\theta|\theta) + \frac{\mathrm{d}Q_M^\theta}{\mathrm{d}\tau}(1 - H(\omega_M^\theta|\theta)) = -dy_0[1 - H(\omega_0|\theta)]$$

This then gives formulas for both $\frac{\mathrm{d}Q_m^\theta}{\mathrm{d}\tau}$ and $\frac{\mathrm{d}Q_M^\theta}{\mathrm{d}\tau}$ by using $\frac{\mathrm{d}Q_M^\theta}{\mathrm{d}\tau} u''(Q_M^\theta) = \alpha \frac{\mathrm{d}Q_m^\theta}{\mathrm{d}\tau} u''(Q_m^\theta)$.

Define $D_m^\theta = H(\omega_m^\theta|\theta) + \frac{\alpha u''(Q_m^\theta)}{u''(Q_M^\theta)}(1 - H(\omega_M^\theta|\theta))$ and $D_M^\theta = \frac{u''(Q_M^\theta)}{\alpha u''(Q_m^\theta)} D_m^\theta$.

We may now derive a numerically implementable formula for IW_0.

$$IW_0 dy_0 = dy_0 \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 < \underline{\omega}^\theta} \Bigg[\left(1 - \frac{H(\omega_m^\theta|\theta) + 1 - H(\omega_M^\theta|\theta)}{D_m^\theta}\right) \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega$$

$$+ \left(1 - \frac{H(\omega_m^\theta|\theta) + 1 - H(\omega_M^\theta|\theta)}{D_M^\theta}\right) \int_{\omega_M^\theta}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \Bigg] g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

$$+ dy_0 \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \omega_m^\theta)} \Bigg[- \frac{H(\omega_m^\theta|\theta) - H(\omega_0|\theta) + 1 - H(\omega_M^\theta|\theta)}{D_m^\theta} \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

$$+ \int_{\omega_0}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} + \left(1 - \frac{H(\omega_m^\theta|\theta) - H(\omega_0|\theta) + 1 - H(\omega^\theta|\theta)}{D_M^\theta}\right) \int_{\omega_M^\theta}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \Bigg] g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

$$+ dy_0 \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\omega_m^\theta, \omega_M^\theta]} \Bigg[- \frac{1 - H(\omega_M^\theta|\theta)}{D_m^\theta} \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$$

$$+ \left(1 - \frac{1 - H(\omega_M^\theta|\theta)}{D_M^\theta}\right) \int_{\omega_M^\theta}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \Bigg] g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

$$+ dy_0 \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in (\omega_M^\theta, \overline{\omega}^\theta]} \Bigg[- \frac{1 - H(\omega_0|\theta)}{D_m^\theta} \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega$$

$$- \frac{1 - H(\omega_0|\theta)}{D_M^\theta} \int_{\omega_M^\theta}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega + \int_{\omega_0}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \Bigg] g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

Note here that $\dfrac{1-H(\omega_m^\theta|\theta)}{D_M^\theta} < 1$ and if $\omega_0 \in (\omega_M^\theta, \overline{\omega}^\theta]$, then $\dfrac{1-H(\omega_0|\theta)}{D_M^\theta} < 1$.

The last step required to run the simulations is to determine $\overline{\phi} = \displaystyle\int_{\boldsymbol{\theta}} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \frac{\Phi'(v)\, u'}{\lambda}\, h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$.

In the standard Mirrlees (1971) (or Saez (2001)) model, in the absence of income effects, it is equal to 1. Here, its value has to be revised to adjust to the change in transfers.
In order to determine it, let us proceed as usual, i.e., by perturbing $\mathcal{L}$ at the optimum by a small uniform lump-sum increase in taxes τ. This does not impact productive effort decisions but it does impact transfer decisions.

$$\mathcal{L}(\tau) = \int_{\boldsymbol{\theta}} \int_{\omega} \Phi\left(u\left(y(\omega) - T(y(\omega)) - \tau - t(\omega,\theta,\tau) - k\left(\frac{y(\omega)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$$

$$+\lambda \int_{\omega} (T(y(\omega)) + \tau)\, f(\omega)\, \mathrm{d}\omega.$$

Then, we must have $\left.\dfrac{\partial \mathcal{L}}{\partial \tau}\right|_{\tau=0} = 0$.

Thus:

$$\lambda = \int_{\boldsymbol{\theta}} \int_{\omega} \left(1 + \frac{\mathrm{d}t}{\mathrm{d}\tau}\right) \Phi'(v)\; u'\, h(\omega|\boldsymbol{\theta})\, g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$$

And $\overline{\phi} = 1 - \dfrac{1}{\lambda} \displaystyle\int_{\boldsymbol{\theta}} \int_{\omega} \frac{\mathrm{d}t}{\mathrm{d}\tau} \Phi'(v)\; u'\, h(\omega|\boldsymbol{\theta})\, g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$.

$\dfrac{\mathrm{d}t}{\mathrm{d}\tau}$ is determined as above – when tax liability is increased lump-sum for the whole population. Thus,

$$\frac{1}{\lambda} \int_{\boldsymbol{\theta}} \int_{\omega} \frac{\mathrm{d}t}{\mathrm{d}\tau} \Phi'(v)\; u'\, h(\omega|\boldsymbol{\theta})\, g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta} = \int_{\boldsymbol{\theta}} \Bigg[\left(1 - \frac{H(\omega_m^\theta|\theta) + 1 - H(\omega_M^\theta|\theta)}{D_m^\theta}\right) \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega$$

$$+ \left(1 - \frac{H(\omega_m^\theta|\theta) + 1 - H(\omega_M^\theta|\theta)}{D_M^\theta}\right) \int_{\omega_M^\theta}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \Bigg] g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

Total welfare effect. Adding up the direct effect $DW_0 = -\int_{\boldsymbol{\theta}} \int_{\omega_0}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}$ and indirect welfare effect IW_0 gives us four different terms:

For individuals belonging to groups in $\overline{\mathcal{C}}_0$ the total welfare effect is:

$$W_0^1 = -\int_{\boldsymbol{\theta}\, s.t.\, \omega_0 < \underline{\omega}^\theta} \left[\delta_0 Q_m \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega + \int_{\omega_m^\theta}^{\omega_M^\theta} \phi(\omega,\theta)\, h(\omega|\boldsymbol{\theta}) \mathrm{d}\omega + \delta_0 Q_M \int_{\omega_M^\theta}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \right] g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

For beneficiaries belonging to groups in $\mathcal{C}_0$ the total welfare effect is:

$$W_0^2 = -\int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \omega_m^\theta)} \left[\delta_0 Q_m \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega + \int_{\omega_m^\theta}^{\omega_M^\theta} \phi(\omega,\theta)\, h(\omega|\boldsymbol{\theta}) \mathrm{d}\omega + \delta_0 Q_M \int_{\omega_M^\theta}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \right] g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

For neutral individuals belonging to groups in $\mathcal{C}_0$ the total welfare effect is:

$$W_0^3 = -\int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\omega_m^\theta, \omega_M^\theta]} \left[\delta_0 Q_m \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega + \int_{\omega_0}^{\omega_M^\theta} \phi(\omega,\theta)\, h(\omega|\boldsymbol{\theta}) \mathrm{d}\omega + \delta_0 Q_M \int_{\omega_M^\theta}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \right] g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

For donors belonging to groups in $\mathcal{C}_0$ the total welfare effect is:

$$W_0^4 = -\int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in (\omega_M^\theta, \overline{\omega}^\theta]} \left[\delta_0 Q_m \int_{\underline{\omega}^\theta}^{\omega_m^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega + \delta_0 Q_M \int_{\omega_M^\theta}^{\overline{\omega}^\theta} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \right] g(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

where $\delta_0 Q_m$ and $\delta_0 Q_M$ are the variations in Q_m and Q_M induced by an infinitesimal increase in all their components from $Y(\omega_0)$ onwards.

This enables to better understand the different effects taking place, and to highlight two kinds of effects: a multiplicative effect and an additive one.

Note first that the effect on donors and receivers is now weighted by the variation of their net consumption with respect to a small variation in all net incomes for skill levels greater than ω_0 (which is positive, so that we are still in the presence of a welfare cost: $DW_0 + IW_0 < 0$).

Appendix C: Proofs

Proof of Proposition 1. This proof can be made in multiple steps.

(i) As in Arrow (1981), suppose that individual i makes a transfer to individual j, who in turn makes a transfer to individual k. Then, from (2) (first oder condition defining transfers),

$$u_c(c_i, y_i, \omega_i) = \alpha u_c(c_j, y_j, \omega_j) \quad \text{and} \quad u_c(c_j, y_j, \omega_j) = \alpha u_c(c_k, y_k, \omega_k)$$

Then: $u_c(c_i, y_i, \omega_i) = \alpha^2 u_c(c_k, y_k, \omega_k)$.
But from (2), we also have that $u_c(c_i, y_i, \omega_i) \geq \alpha u_c(c_k, y_k, \omega_k)$.
So that $\alpha \geq 1$, which is a contradiction.

(ii) Let us place ourselves in a group.

We show this for transfers from individuals of lower ability to individuals of higher ability.
Wlog, suppose that individual 1 is of ability ω_1 and individual 2 is of ability ω_2, such that $\omega_1 < \omega_2$ and that individual 1 makes a transfer to individual 2.

Then, from (i), 1 is a donor and 2 is a receiver (remember that an individual can never be both simultaneously). Let S_1 be the set of individuals 1 makes transfers to, and S_2 the set of individuals who make transfers to 2.
Then from (2), $u'\left(c_1 - k\left(\frac{y_1}{\omega_1}\right)\right) = \alpha u'\left(c_2 - k\left(\frac{y_2}{\omega_2}\right)\right)$, so that $u'\left(c_1 - k\left(\frac{y_1}{\omega_1}\right)\right) < u'\left(c_2 - k\left(\frac{y_2}{\omega_2}\right)\right)$, and since u is concave, $c_1 - k\left(\frac{y_1}{\omega_1}\right) < c_2 - k\left(\frac{y_2}{\omega_2}\right)$.

So:

$$y_1 - t_{12} - \sum_{\substack{i\neq 2\\ i\in U_1}} t_{1i} - k\left(\frac{y_1}{\omega_1}\right) > y_2 + t_{12} + \sum_{\substack{j\neq 2\\ j\in U_2}} t_{j2} - k\left(\frac{y_2}{\omega_2}\right)$$

and

$$2t_{12} < \left(y_1 - k\left(\frac{y_1}{\omega_1}\right)\right) - \left(y_2 - k\left(\frac{y_2}{\omega_2}\right)\right) - \sum_{\substack{i\neq 2\\ i\in U_1}} t_{1i} - \sum_{\substack{j\neq 2\\ j\in U_2}} t_{j2} \quad (*)$$

Now from (1), we know that the incomes of all individuals of skill ω_2 are the same, and all incomes of individuals of ability ω_1 are the same. And that since $\omega_1 < \omega_2$, $y_2 > y_1$.

Suppose that $y_1 - k\left(\frac{y_1}{\omega_1}\right) > y_2 - k\left(\frac{y_2}{\omega_2}\right) \quad (**)$.

Now, since $\omega_2 > \omega_1$, $k\left(\frac{y_1}{\omega_1}\right) > k\left(\frac{y_2}{\omega_2}\right)$.

So $(**)$ implies that $y_1 - k\left(\frac{y_1}{\omega_2}\right) > y_2 - k\left(\frac{y_2}{\omega_2}\right) \quad (***)$.

Now the function $y \mapsto y - k\left(\frac{y}{\omega_2}\right)$ is increasing over $[0, y_2]$, since its derivative cancels at y_2 and since k is convex, so that $y_1 - k\left(\frac{y_1}{\omega_2}\right) \leq y_2 - k\left(\frac{y_2}{\omega_2}\right)$, and $(***)$, thus $(**)$ do not hold.

So that $(*)$ implies that $t_{12} \leq 0$, a contradiction.

The argument is even more straightforward with two individuals of same ability, since we know that they earn the same income.

We show (iii) and (iv) through the characterization of incomes and transfers.

Income. First, the first order condition defining the income of individual i stems from their optimization problem. It is such that $\frac{1}{\omega_i} k\left(\frac{y}{\omega_i}\right) = 1$. This implicitly defines optimal incomes $y_i = y^*(\omega_i)$.

Then, from the first order condition, and since k is decreasing in ω, $\omega \mapsto y^*(\omega) - k\left(\frac{y^*(\omega)}{\omega}\right)$ is increasing in ω. Indeed, from the envelope theorem, its derivative is equal to $\frac{y^*(\omega)}{\omega^2} k'\left(\frac{y^*(\omega)}{\omega}\right) > 0$.

Transfers. Second, let us turn to transfers. Let C be a group, and ω_m (resp. ω_M) the lowest (resp. highest) skill level in this group. And $y_m = y^*(\omega_m)$, $y_M = y^*(\omega_M)$.

Necessary and sufficient condition for transfers to take place.

Necessary condition. Suppose that within group C, $u'\left(y_M - k\left(\frac{y_M}{\omega_M}\right)\right) \geq \alpha u'\left(y_m - k\left(\frac{y_m}{\omega_m}\right)\right)$.

Now suppose that individual j makes a transfer to individual i. Then, from Corollary 1, $y_M \geq y_j > y_i \geq y_m$. Hence,

$$u'\left(y_j - k\left(\frac{y_j}{\omega_j}\right)\right) \geq u'\left(y_M - k\left(\frac{y_M}{\omega_M}\right)\right) \geq \alpha u'\left(y_m - k\left(\frac{y_m}{\omega_m}\right)\right) \geq \alpha u'\left(y_i - k\left(\frac{y_i}{\omega_i}\right)\right)$$

Denote by $t_j > 0$ the total sum of transfers made by j and by t_i the total sum of transfers received by i. Then, the first order condition tells us that

$$u'\left(y_j - t_j - k\left(\frac{y_j}{\omega_j}\right)\right) = \alpha u'\left(y_i + t_i - k\left(\frac{y_i}{\omega_i}\right)\right)$$

So that $u'\left(y_j - k\left(\frac{y_j}{\omega_j}\right)\right) < \alpha u'\left(y_i - k\left(\frac{y_i}{\omega_i}\right)\right)$, which is a contradiction.

Sufficient condition. Suppose that within group C, $u'\left(y_M - k\left(\frac{y_M}{\omega_M}\right)\right) < \alpha u'\left(y_m - k\left(\frac{y_m}{\omega_m}\right)\right)$.

Then, from the first order conditions of the individual problem, this cannot be an equilibrium without transfers. Let us determine the shape of transfers.

Thus, $u'\left(y_M - k\left(\frac{y_M}{\omega_M}\right)\right) < \alpha u'\left(y_m - k\left(\frac{y_m}{\omega_m}\right)\right)$ is a necessary and sufficient condition for transfers to take place in group C. Let us name this condition (T).

Characterization.

Step 1. We know from (i) that individuals are either beneficiaries, donors or neither.

Step 2. Let us show that the after transfer net consumption of all beneficiaries is the same whatever their skill level, and that of all donors is the same whatever their skill level.

Suppose individuals j and m are donors and give to a same individual i. Denote by t_j (resp. t_m) transfers made by j (resp. m), and by t_i transfers received by i. Then, the first order condition tells us that $u'\left(y_j - t_j - k\left(\frac{y_j}{\omega_j}\right)\right) = \alpha u'\left(y_i + t_i - k\left(\frac{y_i}{\omega_i}\right)\right) = u'\left(y_m - t_m - k\left(\frac{y_m}{\omega_j}\right)\right)$.
Hence, individuals j and m have the same consumption net of income.

Now suppose individual j gives to individuals i and m to l. Again, denote by t_j (resp. t_m) total transfers made by j (resp. by m), and by t_i (resp. t_l) total transfers received by i (resp. by l). Then, $u'\left(y_j - t_j - k\left(\frac{y_j}{\omega_j}\right)\right) \geq \alpha u'\left(y_l + t_l - k\left(\frac{y_l}{\omega_l}\right)\right) = u'\left(y_m + t_m - k\left(\frac{y_m}{\omega_m}\right)\right)$.
And in the same way, $u'\left(y_m - t_m - k\left(\frac{y_m}{\omega_m}\right)\right) \geq \alpha u'\left(y_i + t_i - k\left(\frac{y_i}{\omega_i}\right)\right) = u'\left(y_j + t_j - k\left(\frac{y_j}{\omega_j}\right)\right)$.
Hence, $u'\left(y_j - t_j - k\left(\frac{y_j}{\omega_j}\right)\right) = u'\left(y_m + t_m - k\left(\frac{y_m}{\omega_m}\right)\right)$.

So that donors all have the same net consumption.
In the same way, we can show that beneficiaries all have the same net consumption.

Step 3. Let D be the set of donors and B the set of beneficiaries. Then, we know all donors (resp. beneficiaries) have the same after transfer consumption net of cost of effort, which we may denote Q_M (resp. Q_m).
Let $i \in D$ make total transfers t_i. Then, $y_i - t_i - k\left(\frac{y_i}{\omega_i}\right) = Q_M$.
Let $j \in B$ receive total transfers t_j. Then, $y_j + t_j - k\left(\frac{y_j}{\omega_j}\right) = Q_m$.
By definition, $\sum_{i \in D} t_i = \sum_{j \in B} t_j$.
Hence, Q_m and Q_M satisfy the following equation:

$$\sum_{i \in D}\left(y_i - k\left(\frac{y_i}{\omega_i}\right) - Q_M\right) = \sum_{j \in B}\left(Q_m - \left(y_j - k\left(\frac{y_j}{\omega_j}\right)\right)\right)$$

Moreover, Q_M is such that $u'(Q_M) = \alpha u'(Q_m)$ (by the first order conditions, it is the relation satisfied by the marginal utility of a given donor to that of a given beneficiary they make a transfer to).
Moreover, since $t_i > 0$, $y_i - k\left(\frac{y_i}{\omega_i}\right) > Q_M$, and since $t_j > 0$, $y_j - k\left(\frac{y_j}{\omega_j}\right) < Q_m$.
Hence, if it exists, the equilibrium is such that:

$$\sum_{i \text{ s.t. } y_i - k\left(\frac{y_i}{\omega_i}\right) \geq Q_M}\left(y_i - k\left(\frac{y_i}{\omega_i}\right) - u'^{-1}(\alpha u'(Q_m))\right) = \sum_{j \text{ s.t. } y_j - k\left(\frac{y_j}{\omega_j}\right) \leq Q_m}\left(Q_m - \left(y_j - k\left(\frac{y_j}{\omega_j}\right)\right)\right) \tag{3}$$

Existence and uniqueness.

Step 4. Equation (3) has a solution in $(y_m - k\left(\frac{y_m}{\omega_m}\right), y_M - k\left(\frac{y_M}{\omega_M}\right))$.

Let $i = 1, ..., n$ be the individuals in a group C, ordered such that $\omega_1 \leq \omega_2 \leq ... \leq \omega_n$. And suppose that they satisfy condition (T) for transfers to take place, that is

$$u'\left(y_n - k\left(\frac{y_n}{\omega_n}\right)\right) < \alpha u'\left(y_1 - k\left(\frac{y_1}{\omega_1}\right)\right).$$

As in Arrow (1981), define the following function

$$^+ : x \mapsto \begin{cases} x & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x \leq 0 \end{cases}$$

This then allows us to define the following function

$$\Gamma_\alpha(x) = \sum_i \left(y_i - k\left(\frac{y_i}{\omega_i}\right) - u'^{-1}(\alpha u'(x))\right)^+ - \sum_i \left(x - \left(y_i - k\left(\frac{y_i}{\omega_i}\right)\right)\right)^+$$

Γ_α's roots will then be solutions to (3).
Now, $x \mapsto \sum_i \left(x - \left(y_i - k\left(\frac{y_i}{\omega_i}\right)\right)\right)^+$ is increasing in x.

And since u is concave, $u'^{-1}(\alpha u'(.))$ is increasing.
So $x \mapsto \sum_i \left(y_i - k\left(\frac{y_i}{\omega_i}\right) - u'^{-1}(\alpha u'(x))\right)^+$ is decreasing in x.

Hence, Γ_α is decreasing over $\mathbb{R}$.

Moreover, $\sum_i \left((y_1 - k(y_1, \omega_1) - \left(y_i - k\left(\frac{y_i}{\omega_i}\right)\right)\right)^+ = 0$, so that $\Gamma_\alpha(y_1 - k\left(\frac{y_1}{\omega_1}\right)) \geq y_n - k\left(\frac{y_n}{\omega_n}\right) - u'^{-1}(\alpha u'(y_1 - k\left(\frac{y_1}{\omega_1}\right))) \geq 0$ from (T).

$\sum_i \left((y_i - k\left(\frac{y_i}{\omega_i}\right) - \left(y_n - k\left(\frac{y_n}{\omega_n}\right)\right)\right)^+ = 0$, so that $\Gamma_\alpha(u'^{-1}(\frac{1}{\alpha}u'(y_n - k\left(\frac{y_n}{\omega_n}\right)))) \leq u'^{-1}(\frac{1}{\alpha}u'(y_n - k\left(\frac{y_n}{\omega_n}\right))) - (y_1 - k\left(\frac{y_1}{\omega_1}\right)) < 0$ from (T).

Now, because $x \mapsto x$, $x \mapsto u'^{-1}(\alpha u'(x))$ and $^+$ are continuous, so is F.

Hence, from the intermediate value theorem, $\Gamma_\alpha(x) = 0$ has a unique solution, which is in $(y_1 - k\left(\frac{y_1}{\omega_1}\right), u'^{-1}(\frac{1}{\alpha}u'(y_n - k\left(\frac{y_n}{\omega_n}\right))))$.

Moreover, this solution is such that $Q_m > y_1 - k\left(\frac{y_1}{\omega_1}\right)$ and $Q_M < y_n - k\left(\frac{y_1}{\omega_1}\right)$

This equilibrium is a Nash Equilibrium: no individual has an incentive to deviate. Let us consider the three different types in a group C and consider their potential deviations.

First consider individuals i such that $y_i - k\left(\frac{y_i}{\omega_i}\right) \geq Q_M$. These individuals may deviate by giving less, or by giving to individuals whose net income is between Q_M and Q_m.
They may give $\varepsilon > 0$ ($\varepsilon \ll 1$) less to individuals with net income lower than Q_m, so that at the first order, this would result in a change of their utility:

$$\varepsilon(u'(Q_M) - \alpha u'(Q_m)) = 0.$$

This would also be the case if they decided to give $\varepsilon > 0$ less to individuals with net income lower than Q_m.
They may start giving $\varepsilon > 0$ to an individual j with net income between Q_m and Q_M, which would result in a change of their utility:

$$\varepsilon(-u'(Q_M) + \alpha u'(y_j + k\left(\frac{y_j}{\omega_j}\right))) < 0.$$

They may start giving $\varepsilon > 0$ to other individuals with net income greater than Q_M, and their change in utility would be:

$$\varepsilon(-u'(Q_M) + \alpha u'(Q_M)) < 0 < 0.$$

Hence, no individual with net income greater than Q_M has an incentive to deviate from this equilibrium.

We proceed in the same way for the other types of individuals: individuals with net income lower than Q_m may start giving to individuals with net income lower than Q_m, between Q_m and Q_M or above Q_M. And individuals with net income between Q_m and Q_M may do the same. It is straightforward to show that none of these deviations is profitable.

Comparative statics: inequality decreases with α. Let us show that Q_m increases with α and Q_M decreases with α.

Indeed: suppose $\alpha_1 > \alpha_2$. Then: $\forall x$, $u'^{-1}(\alpha_1 u'(x)) < u'^{-1}(\alpha_2 u'(x))$.

So that if $y_i - k\left(\frac{y_i}{\omega_i}\right) > u'^{-1}(\alpha_2 u'(x))$, then $y_i - k\left(\frac{y_i}{\omega_i}\right) > u'^{-1}(\alpha_1 u'(x))$.

So $\forall x$, $\sum_i \left((y_i - k\left(\frac{y_i}{\omega_i}\right) - u'^{-1}(\alpha_2 u'(x))\right)^+ \leq \sum_i \left((y_i - k\left(\frac{y_i}{\omega_i}\right) - u'^{-1}(\alpha_1 u'(x))\right)^+$.

That is, $\Gamma_{\alpha_1}(x) \geq \Gamma_{\alpha_2}(x)$.

And so $Q_m(\alpha_1) > Q_m(\alpha_2)$.

Hence, since net income does not depend on altruism, transfers received by individuals increase. And hence, the transfers made by at least one individual must increase. So Q_M decreases with α.

Proof of Application 1. We use the following specifications for our utility functions:

$$u\left(c - k\left(\frac{y}{\omega}\right)\right) \quad \text{such that} \quad u(x) = \frac{x^{1-\eta}}{1-\eta} \quad \text{and} \quad k\left(\frac{y}{\omega}\right) = \frac{\left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}.$$

We know that from (1), it follows that $y_H = \omega_H^{1+\epsilon}$ and $y_L = \omega_L^{1+\epsilon}$.

In each group, we also know from Proposition 1 (iii) that an equilibrium is either one where all high skilled individuals make transfers to low skilled individuals or where no transfers take place.

Suppose we are in an equilibrium where no transfer takes place. Then by definition, this means that it is not worth deviating for any individual. The only deviation that could take place would be for a high skilled individual to make a transfer to a low skilled one (or to many low skilled individuals). This would imply that the social utility of this high-skilled individual is increasing at $t = 0$.
That is

$$-u'\left(y_H - k\left(\frac{y_H}{\omega_H}\right)\right) + \alpha u'\left(y_L - k\left(\frac{y_L}{\omega_L}\right)\right) > 0$$

i.e. $\alpha^{1/\eta} > \left(\frac{\omega_L}{\omega_H}\right)^{1+\epsilon}$.

So if $\alpha^{1/\eta} \leq \left(\frac{\omega_L}{\omega_H}\right)^{1+\epsilon}$ then the no-transfer situation is a Nash equilibrium.

On the contrary, if $\alpha^{1/\eta} > \left(\frac{\omega_L}{\omega_H}\right)^{1+\epsilon}$ then we are in a situation in which all individuals of high types make the same total transfer t to individuals of low type. Individuals of low type receive $\frac{\pi}{1-\pi}t$ each.

Hence, if i is of high type,

$$c_i = c_H = y_H - t$$

And if j is of low type,

$$c_j = c_L = y_L + \frac{\pi}{1-\pi}t.$$

Transfers are then determined as the solution to

$$u'\left(y_H - t - k\left(\frac{y_H}{\omega_H}\right)\right) = \alpha u'\left(y_L + \frac{\pi}{1-\pi}t - k\left(\frac{y_L}{\omega_L}\right)\right)$$

Which gives us: $t = \dfrac{1}{1+\epsilon}\dfrac{\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{\alpha^{1/\eta} + \frac{\pi}{1-\pi}}$.

The gap in consumption levels in group C^π is then: $c_H - c_L = y_H - y_L - (t + \dfrac{\pi}{1-\pi}t) = y_H - y_L - \dfrac{1}{1-\pi}t$.

So that since y_L and y_H do not depend on π, $\dfrac{\partial}{\partial\pi}(c_H - c_L) = \dfrac{\partial}{\partial\pi}(-\dfrac{1}{1-\pi}t) > 0$.

The gap in consumption levels is higher in groups with higher proportions of high ability individuals.

Proof of Proposition 2. Let us restate the problem of the social planner.

The social planner seeks to maximize the following social welfare function with respect to τ and R:

$$\int_{\boldsymbol{\theta}}\int_{\omega} z(\omega,\boldsymbol{\theta})\Phi(u((1-\tau)y(\omega,\tau) - t(\omega,\boldsymbol{\theta},\tau,R) + R - k\left(\frac{y(\omega,\tau)}{\omega}\right))\, h(\omega|\boldsymbol{\theta})g(\boldsymbol{\theta})\,\mathrm{d}\omega\mathrm{d}\boldsymbol{\theta}$$

under the (government budget) constraint that

$$\tau\int_{\omega} y(\omega)\, f(\omega)\,\mathrm{d}\omega \geq R + B.$$

(At the optimum, the budget constraint is binding.)

And taking into account the game played between individuals.

<u>Preliminary remarks.</u>

As defined in the core of the text, $\mathcal{Y}(\tau) := \displaystyle\int_{\omega} y(\omega)\, f(\omega)\,\mathrm{d}\omega$ is the aggregate income, and enables

us to define $e = \dfrac{(1-\tau)}{\mathcal{Y}} \dfrac{\mathrm{d}\mathcal{Y}}{\mathrm{d}(1-\tau)}$ the elasticity of aggregate earnings with respect to the net-of-tax rate and to rewrite R as a function of τ: $R(\tau) = \tau\mathcal{Y}(\tau) - B$.

It is immediately clear that both at $\tau = 0$ and $\tau = 1$, the demogrant $R = -B$. Moreover, its derivative is equal to $\mathcal{Y}(\tau) + \tau\dfrac{\mathrm{d}\mathcal{Y}}{\mathrm{d}\tau} = \mathcal{Y}(\tau) - \dfrac{\tau}{1-\tau}e(\tau)\mathcal{Y}(\tau)$. Hence, at $\tau = 0$ it is positive and it is negative at $\tau = 1$. Supposing that equation

$$\frac{\tau}{1-\tau} = \frac{1}{e(\tau)}$$

only has one solution (which we name $\overline{\tau}$ in the core of the text), then R's derivative cancels only once, and R is inverse U-shaped.

Here, $t(\omega, \boldsymbol{\theta}, \tau, R)$ represents net transfers, and is positive if individual $(\omega, \boldsymbol{\theta})$ makes transfers, negative if they receive transfers, and equal to zero if they are neither a donor nor a receiver – all of this, according to the government.

Game definition, existence and uniqueness of a solution.

If the government does not know the size of the groups and may only consider how a representative group of type $\boldsymbol{\theta}$ would behave, then it solves the game by extending the equilibrium solution to a continuum of individuals. Note that it is here that the fact that individuals' social utilities are still defined when the number of individuals in the groups becomes very large is important. This allows the government to extend the game to an arbitrarily large number and hence a continuum of individuals the problem stated in Section 2.

Thus, for each $\boldsymbol{\theta}$ and for all $\tau \in [0, 1)$, the government solves

$$\int_{\omega_M^\theta(x)}^{\overline{\omega}^\theta} \left((1-\tau)y(\omega,\tau) + R(\tau) - k\left(\frac{y(\omega,\tau)}{\omega}\right) - u'^{-1}(\alpha u'(x)) \right) h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega$$

$$= \int_{\underline{\omega}^\theta}^{\omega_m^\theta(x)} \left(x - \left((1-\tau)y(\omega) + R(\tau) - k\left(\frac{y(\omega)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega$$

Note here that by the nature of the problem of the social planner, Q_m^θ and Q_M^θ will depend on $\boldsymbol{\theta}$ and will not be group-specific as such (they cannot depend on the realizations that happen in two groups C_1 and C_2 that have the same $\boldsymbol{\theta}$).

$\omega_M^\theta(x)$ and $\omega_m^\theta(x)$ are defined as follows

$$(1-\tau)y(\omega_M^\theta(x),\tau) + R(\tau) - k\left(\frac{y(\omega_M^\theta(x),\tau)}{\omega_M^\theta(x)}\right) = u'^{-1}(\alpha u'(x))$$

$$(1-\tau)y(\omega_m^\theta(x),\tau) + R(\tau) - k\left(\frac{y(\omega_m^\theta(x),\tau)}{\omega_m^\theta(x)}\right) = x$$

We suppose that $x \mapsto \omega_M^\theta(x)$ and $x \mapsto \omega_m^\theta(x)$ are differentiable.

Let $\Gamma(x) = \Gamma_1(x) + \Gamma_2(x)$ where

$$\Gamma_1(x) := \int_{\omega_M^\theta(x)}^{\overline{\omega}^\theta} \left((1-\tau)y(\omega,\tau) + R(\tau) - k\left(\frac{y(\omega,\tau)}{\omega}\right) - u'^{-1}(\alpha u'(x)) \right) h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega$$

$$\text{and } \Gamma_2(x) := \int_{\underline{\omega}^\theta}^{\omega_m^\theta(x)} \left(x - \left((1-\tau)y(\omega,\tau) + R(\tau) - k\left(\frac{y(\omega,\tau)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega.$$

Then, Γ_1 and Γ_2 are continuous and Γ_1 is decreasing and Γ_2 is increasing.

Wlog, let us show this for Γ_1.

First, because ω_M and u are differentiable in x, so is Γ_1.

Then, by definition of $\omega_M^\theta(x)$, $\Gamma_1'(x) = -\int_{\omega_M^\theta(x)}^{\overline{\omega}^\theta} \alpha u''(x) u'^{-1\prime}(\alpha u'(x))\, h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega < 0$.

Hence Γ has a unique root $Q_m(\tau)$, and in the same way as shown in the proof for Proposition 1, it is between $y^*(\underline{\omega},\tau)(1-\tau) - k\left(\frac{y^*(\underline{\omega},\tau)}{\omega}\right)$ and $y^*(\overline{\omega},\tau)(1-\tau) - k\left(\frac{y^*(\overline{\omega},\tau)}{\omega}\right)$.

We suppose that $\tau \mapsto Q_m(\tau)$ is differentiable.

Finally, we have that transfers made equal transfers received, so

$$\int_{\omega_M^\theta(\tau)}^{\overline{\omega}^\theta} \left((1-\tau)y(\omega,\tau) + R(\tau) - k\left(\frac{y(\omega,\tau)}{\omega}\right) - Q_M^\theta(\tau) \right) h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega$$

$$= \int_{\underline{\omega}^\theta}^{\omega_m^\theta(\tau)} \left(Q_m^\theta(\tau) - \left((1-\tau)y(\omega) + R(\tau) - k\left(\frac{y(\omega)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega,$$

where again, $\omega_M^\theta(\tau)$ and $\omega_m^\theta(\tau)$ are defined as follows

$$(1-\tau)y(\omega_M^\theta(\tau),\tau) + R(\tau) - k\left(\frac{y(\omega_M^\theta(\tau),\tau)}{\omega_M^\theta(\tau)}\right) = u'^{-1}(\alpha u'(Q_m(\tau)))$$

$$(1-\tau)y(\omega_m^\theta(\tau),\tau) + R(\tau) - k\left(\frac{y(\omega_m^\theta(\tau),\tau)}{\omega_m^\theta(\tau)}\right) = Q_m(\tau).$$

So that, using the definition of $\omega_M^\theta(\tau)$ and $\omega_m^\theta(\tau)$ and the first order condition defining individuals' incomes,[43]

[43] For all i, income y_i is such that: $1-\tau = \frac{1}{\omega_i} k\left(\frac{y_i}{\omega_i}\right)$

$$\frac{\mathrm{d}Q_m^\theta}{\mathrm{d}\tau} H(\omega_m^\theta(\tau)|\boldsymbol{\theta}) + \frac{\mathrm{d}Q_M^\theta}{\mathrm{d}\tau} (1 - H(\omega_M^\theta(\tau)|\boldsymbol{\theta}))$$
$$= -\left(\int_{\underline{\omega}^\theta}^{\omega_m^\theta(\tau)} y(\omega,\tau)h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega + \int_{\omega_M^\theta(\tau)}^{\overline{\omega}^\theta} y(\omega,\tau)h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega\right) + \frac{\partial R}{\partial \tau}\left(H(\omega_m^\theta(\tau)|\boldsymbol{\theta}) + 1 - H(\omega_M^\theta(\tau)|\boldsymbol{\theta})\right)$$

Optimal linear income tax rate.

Let us now derive the formula for the optimal linear income tax rate.

The first order condition to the problem of the Social Planner is the following:

$$\frac{\mathrm{d}SWF}{\mathrm{d}\tau} = 0.$$

Hence, using the envelope theorem, this implies that

$$\int_{\boldsymbol{\theta}}\int_{\omega} z(\omega,\boldsymbol{\theta})\left(-y(\omega,\tau) - \frac{\mathrm{d}t}{\mathrm{d}\tau}(\omega,\boldsymbol{\theta},\tau,R) + \frac{\partial R}{\partial \tau}\right)\Phi'(u)u'\,h(\omega|\boldsymbol{\theta})g(\boldsymbol{\theta})\,\mathrm{d}\omega\mathrm{d}\boldsymbol{\theta} = 0.$$

Plugging in the following into the above

$$\frac{\partial R}{\partial \tau} = \tau\frac{\mathrm{d}\mathcal{Y}}{\mathrm{d}\tau} + \mathcal{Y} = -\frac{\tau}{1-\tau}e\mathcal{Y} + \mathcal{Y},$$

yields the optimal linear tax rate formula.

Proof of Lemma 1. Start from any linear tax (τ, R) such that private utilities are equal to

$$u\left((1-\tau)y(\omega,\tau) - t(\omega,\theta,\tau,R) + R - k\left(\frac{y(\omega,\tau)}{\omega}\right)\right)$$

Then, the final net consumption of donors $Q_M^\theta(\tau,R)$, the final net consumption of receivers $Q_m^\theta(\tau,R)$ and the skill levels of the last donor $\omega_M^\theta(\tau,R)$ or the last receiver $\omega_m^\theta(\tau,R)$ are the solutions of the following four equations:

(3) $u'(Q_M) = \alpha u'(Q_m)$

(4) $(1-\tau)y(\omega_M) + R - k\left(\frac{y(\omega_M)}{\omega_M}\right) = Q_M$

(5) $(1-\tau)y(\omega_m) + R - k\left(\frac{y(\omega_m)}{\omega_m}\right) = Q_m$

(6) $$\int_{\omega_M}^{\overline{\omega}^\theta}\left((1-\tau)y(\omega,\tau) + R - k\left(\frac{y(\omega,\tau)}{\omega}\right) - Q_M\right)h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega$$
$$= \int_{\underline{\omega}^\theta}^{\omega_m}\left(Q_m - \left((1-\tau)y(\omega,\tau) + R - k\left(\frac{y(\omega,\tau)}{\omega}\right)\right)\right)h(\omega|\boldsymbol{\theta})\,\mathrm{d}\omega$$

First, let us show that if the utility function exhibits constant absolute risk aversion then

$$\frac{\alpha u''(Q_m)}{u''(Q_M)} = 1 \qquad (7a)$$

Indeed, then have that $\frac{u''(Q_M)}{u'(Q_M)} = \frac{u''(Q_m)}{u'(Q_m)}$.

So, using (3), we have that $\frac{\alpha u''(Q_m)}{u''(Q_M)} = \frac{\alpha u'(Q_m)}{u'(Q_M)} = 1$.

Now, if it exhibits decreasing absolute risk aversion then

$$\frac{\alpha u''(Q_m)}{u''(Q_M)} \geq 1 \qquad (7d)$$

Indeed, we know that: $Q_M > Q_m$, so $\frac{u''(Q_M)}{u'(Q_M)} > \frac{u''(Q_m)}{u'(Q_m)}$.

So, using (3), we have that $\frac{\alpha u''(Q_m)}{u''(Q_M)} > \frac{\alpha u'(Q_m)}{u'(Q_M)} = 1$.

And finally, in the same way, if it exhibits increasing absolute risk aversion, then

$$\frac{\alpha u''(Q_m)}{u''(Q_M)} \geq 1 \qquad (7i)$$

Let us study the variations of $Q_M^\theta(\tau, R)$, $Q_m^\theta(\tau, R)$, $\omega_M^\theta(\tau, R)$ and $\omega_m^\theta(\tau, R)$ with respect to R.

For ease of exposition, since we remain in the same group for the whole proof, we drop the θ superscript.

First, from (3), we have that

$$\frac{\partial Q_M}{\partial R} u''(Q_M) = \alpha \frac{\partial Q_m}{\partial R} u''(Q_m) \qquad (8)$$

Then, differentiating (6) with respect to R using Leibniz's rule, we have that

$$\frac{\partial Q_m}{\partial R} F(y_m) + \frac{\partial Q_M}{\partial R}(1 - F(y_M)) = F(y_m) + 1 - F(y_M) \qquad (9)$$

Now (8) tells us that $\frac{\partial Q_M}{\partial R}$ and $\frac{\partial Q_m}{\partial R}$ both have the same sign. And from (9) that they are both positive: indeed, the whole society gets richer, and so do the final consumption levels of donors and receivers.

Now, plugging in (8) into (9), we get that:

$$\frac{\partial Q_m}{\partial R} = \frac{F(y_m) + 1 - F(y_M)}{F(y_m) + \frac{\alpha u''(Q_m)}{u''(Q_M)}(1 - F(y_M))}$$

and

$$\frac{\partial Q_M}{\partial R} = \frac{F(y_m) + 1 - F(y_M)}{\frac{u''(Q_M)}{\alpha u''(Q_m)} F(y_m) + (1 - F(y_M))}.$$

Finally, differentiating (4) and (5) with respect to R, we have that

$$1 + \frac{y(\omega_M)}{\omega_M^2} \frac{\partial \omega_M}{\partial R} k'\left(\frac{y(\omega_M)}{\omega_M}\right) = \frac{\partial Q_M}{\partial R}$$

and

$$1 + \frac{y(\omega_m)}{\omega_m^2} \frac{\partial \omega_m}{\partial R} k'\left(\frac{y(\omega_m)}{\omega_m}\right) = \frac{\partial Q_m}{\partial R}$$

And since transfers received are equal to $t^B(\omega) = Q_m - \left((1-\tau)y(\omega,\tau) + R - k\left(\frac{y(\omega,\tau)}{\omega}\right)\right)$ and transfers made are equal to $t^D(\omega) = -Q_M + \left((1-\tau)y(\omega,\tau) + R - k\left(\frac{y(\omega,\tau)}{\omega}\right)\right)$, we have that

$$\frac{\partial t^B}{\partial R} = \frac{\partial Q_m}{\partial R} - 1$$

and

$$\frac{\partial t^D}{\partial R} = -\frac{\partial Q_M}{\partial R} + 1.$$

Let us first note that the derivative of transfers with respect to the lump sum R does not depend on the identity of individuals (both derivatives are independent of ω), and:

(i) If the utility function exhibits constant absolute risk aversion, then $\frac{\partial Q_m}{\partial R} = \frac{\partial Q_M}{\partial R} = 1$ and $\frac{\partial \omega_m}{\partial R} = \frac{\partial \omega_M}{\partial R} = 0$. The proportions of beneficiaries and of donors stay constant, and transfers stay the same.

(ii) If the utility function exhibits decreasing absolute risk aversion, then $\frac{\partial Q_m}{\partial R} \leq 1$, $\frac{\partial Q_M}{\partial R} \geq 1$, $\frac{\partial \omega_m}{\partial R} \leq 0$ and $\frac{\partial \omega_M}{\partial R} \geq 0$: both the proportions of beneficiaries and donors decrease. Transfers are crowded out by an increase in the lump-sum transfer made by the government on all individuals.

(iii) If the utility function exhibits decreasing absolute risk aversion, then $\frac{\partial Q_m}{\partial R} \geq 1$, $\frac{\partial Q_M}{\partial R} \leq 1$, $\frac{\partial \omega_m}{\partial R} \geq 0$ and $\frac{\partial \omega_M}{\partial R} \leq 0$: both the proportions of beneficiaries and donors increase. Transfers are crowded in by an increase in the lump-sum transfer made by the government on all individuals.

Proof of Lemma 2. Start again from any linear tax (τ, R) such that private utilities are equal to

$$u\left((1-\tau)y(\omega,\tau) - t(\omega,\theta,\tau,R) + R - k\left(\frac{y(\omega,\tau)}{\omega}\right)\right)$$

Suppose that both the proportions of beneficiaries and of donors increase. Then

$$\frac{\partial \omega_M}{\partial \tau} < 0 \qquad \text{and} \qquad \frac{\partial \omega_M}{\partial \tau} > 0.$$

Differentiating (3), (4) and (5) with respect to τ we get:

$$\frac{\partial Q_M}{\partial \tau} u''(Q_M) = \alpha \frac{\partial Q_m}{\partial \tau} u''(Q_m) \qquad (10)$$

$$-y(\omega_M) + \frac{y(\omega_M)}{\omega_M^2} \frac{\partial \omega_M}{\partial \tau} k'\left(\frac{y(\omega_M)}{\omega_M}\right) = \frac{\partial Q_M}{\partial \tau} \qquad (11)$$

and

$$-y(\omega_m) + \frac{y(\omega_m)}{\omega_m^2} \frac{\partial \omega_m}{\partial \tau} k'\left(\frac{y(\omega_m)}{\omega_m}\right) = \frac{\partial Q_m}{\partial \tau} \qquad (12)$$

Plugging both (11) and (12) into (10), we get:

$$\left(-y(\omega_M) + \frac{y(\omega_M)}{\omega_M^2} \frac{\partial \omega_M}{\partial \tau} k'\left(\frac{y(\omega_M)}{\omega_M}\right)\right) u''(Q_M) = \alpha u''(Q_m)\left(-y(\omega_m) + \frac{y(\omega_m)}{\omega_m^2} \frac{\partial \omega_m}{\partial \tau} k'\left(\frac{y(\omega_m)}{\omega_m}\right)\right) \qquad (13)$$

So that,

$$-y(\omega_M)u''(Q_M) + \alpha y(\omega_m)u''(Q_m) = -\frac{y(\omega_M)}{\omega_M^2} \frac{\partial \omega_M}{\partial \tau} k'\left(\frac{y(\omega_M)}{\omega_M}\right) u''(Q_M) + \alpha u''(Q_m) \frac{y(\omega_m)}{\omega_m^2} \frac{\partial \omega_m}{\partial \tau} k'\left(\frac{y(\omega_m)}{\omega_m}\right)$$

And since we suppose that $\frac{\partial \omega_M}{\partial \tau} < 0$ and $\frac{\partial \omega_M}{\partial \tau} > 0$, we have that

$$-y(\omega_M)u''(Q_M) + \alpha y(\omega_m)u''(Q_m) < 0.$$

Hence,

$$\frac{\alpha u''(Q_m)}{u''(Q_M)} > \frac{y(\omega_M)}{y(\omega_m)}.$$

Now, we have that:

$$Q_m y(\omega_M) \geq Q_M y(\omega_m)$$

So using that $u'(Q_M) = \alpha u'(Q_m)$, we have

$$u'(Q_M) Q_m y(\omega_M) \geq \alpha u'(Q_m) Q_M y(\omega_m)$$

Hence: $\frac{\alpha y(\omega_m)}{y(\omega_M)} \leq \frac{u'(Q_M)/Q_M}{u'(Q_m)/Q_m}$.

Now, since u exhibits constant or increasing relative risk aversion, $x \mapsto \frac{xu''(x)}{u'(x)}$ is non-increasing.

So $\frac{Q_M u''(Q_M)}{u'(Q_M)} \leq \frac{Q_m u''(Q_m)}{u'(Q_m)}$.

Hence, $\frac{u'(Q_M)/Q_M}{u'(Q_m)/Q_m} \leq \frac{u''(Q_M)}{u''(Q_m)}$.

So, $\frac{\alpha y(\omega_m)}{y(\omega_M)} \leq \frac{u''(Q_M)}{u''(Q_m)}$, and $\frac{\alpha u''(Q_m)}{u''(Q_M)} \leq \frac{y(\omega_M)}{y(\omega_m)}$, which is a contradiction.

(In the same way, if $u''' < 0$, then $u''(Q_M) < u''(Q_m)$ so $\frac{\alpha u''(Q_m)}{u''(Q_M)} < 1 < \frac{y(\omega_M)}{y(\omega_m)}$).

So we must have that

$$\frac{\partial \omega_M}{\partial \tau} \geq 0 \qquad \text{or/and} \qquad \frac{\partial \omega_M}{\partial \tau} \leq 0.$$

So, using (11) and (12), we must have that

$$\frac{\partial Q_M}{\partial \tau} + y(\omega_M) \geq 0 \qquad \text{or/and} \qquad \frac{\partial Q_m}{\partial \tau} + y(\omega_m) \leq 0.$$

And so

$$\forall \omega \leq \omega_m, \quad \frac{\partial t^B}{\partial \tau}(\omega) = \frac{\partial Q_m}{\partial \tau} + y(\omega) \leq \frac{\partial Q_m}{\partial \tau} + y(\omega_m) \leq 0,$$

or/and

$$\forall \omega \geq \omega_M, \quad \frac{\partial t^D}{\partial \tau}(\omega) = -\frac{\partial Q_M}{\partial \tau} - y(\omega) \leq -\frac{\partial Q_M}{\partial \tau} - y(\omega_M) \leq 0.$$

So suppose that $\frac{\partial t^B}{\partial \tau}(\omega) \leq 0$ for all $\omega \leq \omega_m$, then $\int_{\underline{\omega}}^{\omega_m} \frac{\partial t^B}{\partial \tau}(\omega)\, h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \leq 0$, so $\int_{\omega_M}^{\overline{\omega}} \frac{\partial t^D}{\partial \tau}(\omega)\, h(\omega|\boldsymbol{\theta})\, \mathrm{d}\omega \leq 0$, and we have the same if $\frac{\partial t^D}{\partial \tau}(\omega) \leq 0$ for all $\omega \geq \omega_M$.

Hence, total transfers made and received are crowded-out by an increase in the marginal tax rate.

Sign of the derivative of transfers. We know that the effect of an increase in the tax rate on transfers is:

$$\frac{\mathrm{d}t^B}{\mathrm{d}\tau} = \frac{\partial t^B}{\partial \tau} + \frac{\partial R}{\partial \tau}\frac{\partial t^B}{\partial R} \quad \text{and} \quad \frac{\mathrm{d}t^D}{\mathrm{d}\tau} = \frac{\partial t^D}{\partial \tau} + \frac{\partial R}{\partial \tau}\frac{\partial t^D}{\partial R}$$

We are thus interested in the sign of $\frac{\partial R}{\partial \tau}$ at the optimum and show that at the optimum, $\frac{\partial R}{\partial \tau} > 0$.

Sign of $\frac{\partial R}{\partial \tau}$.

Let us first show that at the optimum, $\frac{\partial R}{\partial \tau} > 0$.

Suppose that at the optimum $\frac{\partial R}{\partial \tau} \leq 0$.

In that case, in groups where there are no transfers, individuals' utilities are of the form

$$u\left((1-\tau)y + R - k\left(\frac{y}{\omega}\right)\right).$$

Now, using the individuals' first order condition determining their labor supply, we have that at the optimum

$$\left(-y + \frac{\partial R}{\partial \tau}\right) u'\left((1-\tau)y + R - k\left(\frac{y}{\omega}\right)\right) < 0.$$

So that the government could decrease τ by an infinitesimal amount and increase individuals' utility levels.

In that case, in groups of characteristic $\boldsymbol{\theta}$ where according to the government transfers take place,

using (4) we have that $\frac{\mathrm{d}Q_m^\theta}{\mathrm{d}\tau}$, $\frac{\mathrm{d}Q_M^\theta}{\mathrm{d}\tau} < 0$. Indeed, since $Q_m^\theta = u'^{-1}(\alpha u'(Q_M^\theta))$, both derivatives have the same sign. Then, donors have utility $u(Q_M^\theta)$, beneficiaries have utility $u(Q_m^\theta)$, and

$$\frac{\mathrm{d}Q_m^\theta}{\mathrm{d}\tau} u'(Q_m^\theta) < 0 \quad \text{and} \quad \frac{\mathrm{d}Q_M^\theta}{\mathrm{d}\tau} u'(Q_M^\theta) < 0.$$

The government could thus decrease τ by an infinitesimal amount and increase both donors' and beneficiaries' utility levels. Neutral individuals have utility $u\left((1-\tau)y + R - k\left(\frac{y}{\omega}\right)\right)$. With the same argument as above, the government could decrease τ by an infinitesimal amount and increase neutral individuals' utility levels.

Hence, it cannot be that at the optimum $\frac{\partial R}{\partial \tau} \leq 0$. □

Proof of Example 1 (Linear income tax.) Using the functional specifications defined above, our first order conditions defining the labor supply individuals choose imply that $y_H = (1-\tau)^\epsilon \omega_H^{1+\epsilon}$ and $y_L = (1-\tau)^\epsilon \omega_L^{1+\epsilon}$.

For ease of exposure, suppose that $B = 0$. In this case the tax is a purely redistributive one.

Then, $R(\tau) = \tau(1-\tau)^\epsilon \int_0^1 \left(\pi\omega_H^{1+\epsilon} + (1-\pi)\omega_L^{1+\epsilon}\right) f(\pi)\,\mathrm{d}\pi$,

which may also be rewritten $R(\tau) = \tau(1-\tau)^\epsilon(\mu_\pi \omega_H^{1+\epsilon} + (1-\mu_\pi)\omega_L^{1+\epsilon})$, where $\mu_\pi := \mathbb{E}(\pi)$.

Then, following a similar reasoning to that in the *laissez-faire* case, a Nash equilibrium with transfers takes place if and only if:

$$\tau < \frac{\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{(\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) + (1-\alpha^{1/\eta})(1+\epsilon)(\mu_\pi\omega_H^{1+\epsilon} + (1-\mu_\pi)\omega_L^{1+\epsilon})}.$$

Let us note also that since $\tau \in [0,1]$ this is only possible if we have the same condition as under the *laissez-faire*; that is, $\alpha^{1/\eta} > \left(\frac{\omega_L}{\omega_H}\right)^{1+\epsilon}$. This remains a necessary condition.

Let us denote $\overline{\tau} = \frac{\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}}{(\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) + (1-\alpha^{1/\eta})(1+\epsilon)(\mu_\pi\omega_H^{1+\epsilon} + (1-\mu_\pi)\omega_L^{1+\epsilon})}$.

Then we will have an equilibrium with transfers if $\tau^* < \overline{\tau}$, and a no-transfer equilibrium if $\tau^* \geq \overline{\tau}$.

Let us now show that at the optimum, $\frac{\partial R}{\partial \tau}(\tau^*) \geq 0$.

Suppose that $\tau^* \geq \overline{\tau}$.
Then, there are no transfers taking place, and

$$u\left((1-\tau)y_H + R(\tau) - k(\frac{y_H}{\omega_H}\right) = u\left(\frac{\omega_H^{1+\epsilon}}{1+\epsilon}(1-\tau)^{1+\epsilon} + R(\tau)\right)$$

and

$$u\left((1-\tau)y_L + R(\tau) - k(\frac{y_L}{\omega_L}\right) = u\left(\frac{\omega_L^{1+\epsilon}}{1+\epsilon}(1-\tau)^{1+\epsilon} + R(\tau)\right)$$

Then, $\frac{\partial R}{\partial \tau}(\tau^*) < 0$, since u is increasing and $\tau \mapsto (1-\tau)^{1+\epsilon}$ is decreasing in τ, the government could reduce the tax rate by a small increment, increase all individuals' utility levels as well as revenue – a contradiction.

Suppose now that there are transfers taking place, i.e., $\tau^* < \overline{\tau}$.
Then, transfers are defined by the following equation:

$$((1-\tau)y_H - t + R(\tau) - k\left(\frac{y_H}{\omega_H}\right))^{-\eta} \geq \alpha\left((1-\tau)y_L + \frac{\pi}{1-\pi}t + R(\tau) - k\left(\frac{y_L}{\omega_L}\right)\right)^{-\eta},$$

so that: $t = \dfrac{\frac{(1-\tau)^{1+\epsilon}}{1+\epsilon}(\alpha^{1/\eta}\omega_H^{1+\epsilon} - \omega_L^{1+\epsilon}) - R(\tau)(1-\alpha^{1/\eta})}{\alpha^{1/\eta} + \frac{\pi}{1-\pi}}$

And

$$u\left((1-\tau)y_H + R(\tau) - t - k\left(\frac{y_H}{\omega_H}\right)\right) = u\left(\frac{\frac{(1-\tau)^{1+\epsilon}}{1+\epsilon}(\pi\omega_H^{1+\epsilon} + (1-\pi)\omega_L^{1+\epsilon}) + R(\tau)}{\alpha^{1/\eta}(1-\pi)+\pi}\right)$$

and

$$u\left((1-\tau)y_L + R(\tau) + \frac{\pi}{1-\pi}t - k\left(\frac{y_L}{\omega_L}\right)\right) = u\left(\alpha^{1/\eta}\frac{\frac{(1-\tau)^{1+\epsilon}}{1+\epsilon}(\pi\omega_H^{1+\epsilon} + (1-\pi)\omega_L^{1+\epsilon}) + R(\tau)}{\alpha^{1/\eta}(1-\pi)+\pi}\right)$$

Then, with exactly the same arguments as above, it cannot be that at the optimum, $\frac{\mathrm{d}R}{\mathrm{d}\tau}(\tau^*) < 0$.

So at the optimum, $\frac{\mathrm{d}R}{\mathrm{d}\tau}(\tau^*) \geq 0$.

Hence, since $R(\tau) = \tau(1-\tau)^{\epsilon}(\mu_\pi\omega_H^{1+\epsilon} + (1-\mu_\pi)\omega_L^{1+\epsilon})$, this is equivalent to

$$\tau^* \leq \frac{1}{1+\epsilon}$$

This enables us, also, to know the sign of $\frac{\mathrm{d}t}{\mathrm{d}\tau}(\pi,\tau)$ at the optimum. Indeed, since at the optimum $\frac{\mathrm{d}R}{\mathrm{d}\tau}(\tau^*) \geq 0$, $\frac{\mathrm{d}t}{\mathrm{d}\tau}(\pi,\tau^*) < 0$.

Proof of Proposition 3. For ease of exposure we write the Lagrangian for the Social Planner's problem with no Pareto weights:[44]

$$\mathcal{L} = \int_{\boldsymbol{\theta}}\int_{\omega} \Phi\left(u\left(y - T(y) - t - k\left(\frac{y}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta})g(\boldsymbol{\theta})\,\mathrm{d}\omega\mathrm{d}\boldsymbol{\theta} + \lambda\int_{\omega} T(y(\omega))\,f(\omega)\,\mathrm{d}\omega.$$

[44]This is straightforwardly with no loss of generality.

Now introduce a small tax perturbation around y_0: the perturbed tax schedule is of the form:

$$T_\tau(y) = \begin{cases} T(y) & \text{if} \quad y \leq y_0 \\ T(y) + \tau(y - y_0) & \text{if} \quad y_0 \leq y \leq y_0 + dy_0 \\ T(y) + \tau dy_0 & \text{if} \quad y \geq y_0 + dy_0 \end{cases}$$

Let ω_0 be the productivity level corresponding to earning level y_0 under the tax schedule $T(.)$. Then, under the perturbed tax schedule, since there are no income effects, individuals with productivity ω_0 still earn y_0. Define $\omega_0 + d\omega_0$ the productivity level such that $y(\omega_0 + d\omega_0) = y_0 + dy_0$ under the perturbed tax schedule.

Then, $\mathcal{L}$ depends on τ and on λ. Let us rewrite it as follows: $\mathcal{L}(\tau, \lambda) = \mathcal{L}^A(\tau, \lambda) + \mathcal{L}^B(\tau, \lambda) + \mathcal{L}^C(\tau, \lambda)$, with:

$$\mathcal{L}^A = \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 > \overline{\omega}^\theta} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \left[\Phi\left(u\left(y(\omega) - T_\tau(y(\omega)) - t(\omega, \theta) - k\left(\frac{y(\omega)}{\omega} \right) \right) \right) + \lambda T_\tau(y(\omega)) \right] h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$$

$$\mathcal{L}^B = \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \left[\Phi\left(u\left(y(\omega) - T_\tau(y(\omega)) - t(\omega, \theta) - k\left(\frac{y(\omega)}{\omega} \right) \right) \right) + \lambda T_\tau(y(\omega)) \right] h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$$

$$\mathcal{L}^C = \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 < \underline{\omega}^\theta} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \left[\Phi\left(u\left(y(\omega) - T_\tau(y(\omega)) - t(\omega, \theta) - k\left(\frac{y(\omega)}{\omega} \right) \right) \right) + \lambda T_\tau(y(\omega)) \right] h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$$

Now we immediately have that $\dfrac{\partial \mathcal{L}^A}{\partial \tau} = 0$.

$\dfrac{\partial \mathcal{L}^C}{\partial \tau}$ will bring out the mechanical, as well as the direct and indirect welfare effects. As there are no income effects, there will be no behavioral term.

More specifically:[45]

$$\frac{\partial \mathcal{L}^C}{\partial \tau} = -\int_{\boldsymbol{\theta}\, s.t.\, \omega_0 < \underline{\omega}^\theta} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \left(\frac{\mathrm{d}t}{\mathrm{d}\tau} + dy_0 \right) \Phi'(v) \times u' \times h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta} + \lambda \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 < \underline{\omega}^\theta} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} dy_0\, h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$$

Finally, let us again separate $\mathcal{L}^B$ into three parts: $\mathcal{L}^B = \mathcal{L}_1^B + \mathcal{L}_2^B + \mathcal{L}_3^B$, where:

$$\mathcal{L}_1^B = \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\underline{\omega}^\theta}^{\omega_0} \left[\Phi\left(u\left(y(\omega) - T_\tau(y(\omega)) - t(\omega, \theta) - k\left(\frac{y(\omega)}{\omega} \right) \right) \right) + \lambda T_\tau(y(\omega)) \right] h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$$

$$\mathcal{L}_2^B = \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\omega_0}^{\omega_0 + d\omega_0} \left[\Phi\left(u\left(y(\omega) - T_\tau(y(\omega)) - t(\omega, \theta) - k\left(\frac{y(\omega)}{\omega} \right) \right) \right) + \lambda T_\tau(y(\omega)) \right] h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$$

[45] For ease of notation we omit, when convenient, the dependence of y on ω and that of t on (ω, θ), and may also refer to u and $\Phi'(v)$ without any further specification.

$$\mathcal{L}_3^B = \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\omega_0 + d\omega_0}^{\overline{\omega}^\theta} \left[\Phi\left(u\left(y(\omega) - T_\tau(y(\omega)) - t(\omega,\theta) - k\left(\frac{y(\omega)}{\omega}\right)\right)\right) + \lambda T_\tau(y(\omega)) \right] h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$$

The agents in the first term only undergo the indirect welfare effect: their taxes are not raised so the government gathers no extra tax revenue on them, nor does it hurt them on a direct welfare point of view. Thus: $\dfrac{\partial \mathcal{L}_1^B}{\partial \tau} = -\displaystyle\int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\underline{\omega}^\theta}^{\omega_0} \frac{\mathrm{d}t}{\mathrm{d}\tau} \times \Phi'(v) \times u' \times h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}.$

The agents in the third term will be similar to agents in term C:

$$\begin{aligned}\frac{\partial \mathcal{L}_3^B}{\partial \tau} = &-\int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\omega_0 + d\omega_0}^{\overline{\omega}^\theta} \left(\frac{\mathrm{d}t}{\mathrm{d}\tau} + dy_0\right) \Phi'(v) \times u' \times h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta} \\ &+ \lambda \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\omega_0 + d\omega_0}^{\overline{\omega}^\theta} dy_0\, h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta}\end{aligned}$$

Finally the term comprising the agents directly affected by the marginal tax increase will generate all four effects:

$$\begin{aligned}\frac{\partial \mathcal{L}_2^B}{\partial \tau} = &-\int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\omega_0}^{\omega_0 + d\omega_0} \left(\frac{\mathrm{d}t}{\mathrm{d}\tau} + dy_0\right) \Phi'(v) \times u' \times h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta} \\ &+ \lambda \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\omega_0}^{\omega_0 + d\omega_0} dy_0\, h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta} \\ &- \lambda \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\omega_0}^{\omega_0 + d\omega_0} \xi^c(\omega) \times y(\omega) \times \frac{T'(y(\omega))}{1 - T'(y(\omega))} h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta},\end{aligned}$$

which will tend to $-\lambda\, d\omega_0 \times e(y_0) \times y_0 \times \dfrac{T'(y_0)}{1 - T'(y_0)} \times f(\omega_0)$ for small τ and small dy_0 (and thus, small $d\omega_0$).

Let $f^*(y)$ be the density at y under a linearized $T(.)$ (as defined in Saez (2001)).

By definition of $f^*(y)$, $f^*(y_0)dy_0 = f(\omega_0)d\omega_0$, so that setting $\dfrac{\partial \mathcal{L}^A}{\partial \tau} + \dfrac{\partial \mathcal{L}^B}{\partial \tau} + \dfrac{\partial \mathcal{L}^C}{\partial \tau} = 0$, then yields the necessary condition on the marginal tax rate for the tax schedule to be optimal at y_0:

$$\frac{T'(y_0)}{1 - T'(y_0)} = \frac{1}{e(y_0)} \times \frac{1}{y_0\, f^*(y_0)} \times \left(\int_{y_0}^{\overline{y}} (1 - \phi(y)) f_y(y)\, \mathrm{d}y \; + \; IW_0 \right)$$

where $IW_0 = -\displaystyle\int_{\boldsymbol{\theta}\, s.t.\, \omega_0 < \underline{\omega}^\theta} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \frac{\mathrm{d}t}{\mathrm{d}\tau} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta} - \int_{\boldsymbol{\theta}\, s.t.\, \omega_0 \in [\underline{\omega}^\theta, \overline{\omega}^\theta]} \int_{\underline{\omega}^\theta}^{\overline{\omega}^\theta} \frac{\mathrm{d}t}{\mathrm{d}\tau} \phi(\omega,\theta) h(\omega|\boldsymbol{\theta}) g(\boldsymbol{\theta})\, \mathrm{d}\omega \mathrm{d}\boldsymbol{\theta},$

Sign of $\delta_0 Q_m - 1$ and $\delta_0 Q_M - 1$. Suppose utilities are CRRA.

u is then of the form: $u(x) = \dfrac{x^{1-\eta}}{1-\eta}$, so that $u'(Q_M) = \alpha u'(Q_m)$ implies $Q_M^{-\eta} = \alpha Q_m^{-\eta}$, i.e., $Q_m = \alpha^{\frac{1}{\eta}} Q_M$.

Then, $\dfrac{\alpha u''(Q_m)}{u''(Q_M)} = \alpha^{-\eta} > 1$.

$\omega_0 < \underline{\omega}^\theta$. From Appendix B, we know that if $\omega_0 < \underline{\omega}^\theta$ then $\delta_0 Q_m = \dfrac{F(\omega_m^\theta) + 1 - F(\omega_M^\theta)}{F(\omega_m^\theta) + \frac{\alpha u''(Q_m^\theta)}{u''(Q_M^\theta)}(1 - F(\omega_M^\theta))}$.

So since $\dfrac{\alpha u''(Q_m)}{u''(Q_M)} = \alpha^{-\eta} > 1$, $\delta_0 Q_m < 1$.

On the other hand, $\delta_0 Q_M = \dfrac{F(\omega_m^\theta) + 1 - F(\omega_M^\theta)}{F(\omega_m^\theta)\frac{u''(Q_M^\theta)}{\alpha u''(Q_m^\theta)} + 1 - F(\omega_M^\theta)}$.

And since $\dfrac{u''(Q_M)}{\alpha u''(Q_m)} = \alpha^{\eta} < 1$, $\delta_0 Q_M > 1$.

$\omega_0 \in [\underline{\omega}^\theta, \omega_m^\theta)$. Then $\delta_0 Q_m = \dfrac{F(\omega_m^\theta) - F(\omega_0) + 1 - F(\omega_M^\theta)}{F(\omega_m^\theta) + \frac{\alpha u''(Q_m^\theta)}{u''(Q_M^\theta)}(1 - F(\omega_M^\theta))}$, so that here also $\delta_0 Q_m < 1$.

The sign of $\delta_0 Q_M - 1$ is undetermined.

$\omega_0 \in (\omega_m^\theta, \omega_M^\theta)$. Then $\delta_0 Q_m = \dfrac{1 - F(\omega_M^\theta)}{F(\omega_m^\theta) + \frac{\alpha u''(Q_m^\theta)}{u''(Q_M^\theta)}(1 - F(\omega_M^\theta))}$, so that again $\delta_0 Q_m < 1$.

Here, we also have that $\delta_0 Q_M < 1$, since $\delta_0 Q_M = \dfrac{1 - F(\omega_M^\theta)}{F(\omega_m^\theta)\frac{u''(Q_M^\theta)}{\alpha u''(Q_m^\theta)} + 1 - F(\omega_M^\theta)}$.

$\omega_0 \in (\omega_m^\theta, \overline{\omega}^\theta]$. Then $\delta_0 Q_m = \dfrac{1 - F(\omega_0)}{F(\omega_m^\theta) + \frac{\alpha u''(Q_m^\theta)}{u''(Q_M^\theta)}(1 - F(\omega_M^\theta))}$ and $\delta_0 Q_M = \dfrac{1 - F(\omega_0)}{F(\omega_m^\theta)\frac{u''(Q_M^\theta)}{\alpha u''(Q_m^\theta)} + 1 - F(\omega_M^\theta)}$.

So that once again $\delta_0 Q_m < 1$ and $\delta_0 Q_M < 1$.

Note that in the two last cases, $\delta_0 Q_M < 1$ does not depend on the form of u. It holds for all functions u that are increasing and concave.

Proof of Proposition 4. Start with the following perturbation.

$$T_\tau(y) = \begin{cases} T(y) & \text{if} \quad y \leq y_0 \\ T(y) + \tau(y - y_0) & \text{if} \quad y_0 \leq y \leq y_0 + dy_0 \\ T(y) + \tau dy_0 & \text{if} \quad y \geq y_0 + dy_0 \end{cases}$$

First take y_0 to be between $\underline{y}_2$ and $\overline{y}_2$, the incomes of individuals in group C_2 under T.

Then,

$$\mathcal{L} = \int_{\underline{\omega}^1}^{\overline{\omega}^1} \Phi\Big(v\Big(y(\omega, \tau^K) - T(y(\omega, \tau^K)) - \tau dy_0 - t^K\Big(y(\omega, \tau^K) - T(y(\omega, \tau^K)) - \tau dy_0 \Big)$$

$$
\begin{aligned}
&-t(\omega,\boldsymbol{\theta}_1,\tau^K,y^K)-k\left(\frac{y(\omega,\tau^K)}{\omega}\right)\Bigg)\Bigg)\,h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega\\
&+\int_{\underline{\omega}^2}^{\omega_0}\Phi\left(v\left(y(\omega)-T(y(\omega))+L^K-t(\omega,\boldsymbol{\theta}_2,L^K)-k\left(\frac{y(\omega)}{\omega}\right)\right)\right)\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega\\
&+\int_{\omega_0}^{\omega_0+d\omega_0}\Phi\left(v\left(y(\omega)-T(y(\omega))+L^K-\tau(y(\omega)-y_0)-t(\omega,\boldsymbol{\theta}_2,L^K)-k\left(\frac{y(\omega)}{\omega}\right)\right)\right)\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega\\
&+\int_{\omega_0+d\omega_0}^{\overline{\omega}^2}\Phi\left(v\left(y(\omega)-T(y(\omega))+L^K-\tau dy_0-t(\omega,\boldsymbol{\theta}_2,L^K)-k\left(\frac{y(\omega)}{\omega}\right)\right)\right)\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega\\
&+\lambda\int_{\underline{\omega}^1}^{\overline{\omega}^1}\left(T(y(\omega,\tau^K))+\tau dy_0\right)\,h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega+\lambda\int_{\underline{\omega}^2}^{\omega_0}T(y(\omega))\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega\\
&+\lambda\int_{\omega_0}^{\omega_0+d\omega_0}\left(T(y(\omega))+\tau(y-y_0)\right)\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega+\lambda\int_{\omega_0+d\omega_0}^{\overline{\omega}^2}\left(T(y(\omega))+\tau dy_0\right)\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega.
\end{aligned}
$$

With $L^K=\displaystyle\int_{\underline{\omega}^1}^{\overline{\omega}^1}t^K\Big(y(\omega,\tau^K)-T(y(\omega,\tau^K))-\tau dy_0\Big)\,h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega.$

Then, using the envelope theorem, for $\tau\to 0$

$$
\begin{aligned}
\frac{d\mathcal{L}}{d\tau}=&\int_{\underline{\omega}^1}^{\omega'^K}\left(-dy_0-\frac{dt}{d\tau}\right)\Phi'(v)\,u'\,h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega\\
&+\int_{\omega'^K}^{\overline{\omega}^1}\left(dy_0\tau^K-dy_0-\frac{dt}{d\tau}\right)\Phi'(v)\,u'\,h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega\\
&+\int_{\underline{\omega}^2}^{\omega_0}\left(\frac{dL^K}{d\tau}-\frac{dt}{d\tau}\right)\Phi'(v)\,u'\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega\\
&+\int_{\omega_0}^{\omega_0+d\omega_0}\left(\frac{dL^K}{d\tau}-\frac{dt}{d\tau}-(y(\omega)-y_0)\right)\Phi'(v)\,u'\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega\\
&+\int_{\omega_0+d\omega_0}^{\overline{\omega}^2}\left(\frac{dL^K}{d\tau}-\frac{dt}{d\tau}-dy_0\right)\Phi'(v)\,u'\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega\\
&+\lambda\int_{\underline{\omega}^1}^{\overline{\omega}^1}dy_0\,h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega\\
&+\lambda\int_{\omega_0}^{\omega_0+d\omega_0}\left(\frac{\partial y}{\partial\tau}T'(y(\omega,\tau^K))+(y-y_0)\right)\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega+\lambda\int_{\omega_0+d\omega_0}^{\overline{\omega}^2}dy_0\,h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega
\end{aligned}
$$

We have: $\dfrac{dL^K}{d\tau}=-dy_0\tau^K\Big(H(\overline{\omega}^1|\boldsymbol{\theta}_1)-H(\omega'^K|\boldsymbol{\theta}_1)\Big)$.

The solution is very similar if y_0 is between $\underline{y}_1$ and $y(\omega'^K)$, the incomes under T of individuals who are not extracted in group C_1.

The behavioural term, however, differs if y_0 is between $y(\omega'^K)$ and $\overline{y}_1$. Indeed, the double distortion then has to be taken into account in the behavioral term. Now, $-e(y_0)=\dfrac{\partial y}{\partial\tau}\dfrac{(1-T'(y_0))(1-\tau^K)}{y_0}$.

Proof of Proposition 5. First take y_0 to be between $\underline{y}_2$ and $\overline{y}_2$, the incomes of individuals in group C_2 under T.

Then,

$$\begin{aligned}
\mathcal{L} = &\int_{\underline{\omega}^1}^{\omega'^K} \Phi\left(v\left(y(\omega,\tau^K) - T(y(\omega,\tau^K)) - \tau dy_0 - t(\omega,\boldsymbol{\theta}_1,\tau^K,\gamma^K,\overline{\omega}^2) - k\left(\frac{y(\omega,\tau^K)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega \\
&+ \int_{\omega'^K}^{\overline{\omega}^1} \Phi\Big(v\Big(y(\omega,\tau^K) - T(y(\omega,\tau^K)) - \tau dy_0 - t(\omega,\boldsymbol{\theta}_1,\tau^K,\gamma^K,\overline{\omega}^2) \\
&- \tau^K\left[y(\omega,\tau^K) - T(y(\omega,\tau^K)) - \tau dy_0 - \gamma^K\left(y(\overline{\omega}^2) - T(y(\overline{\omega}^2)) - \tau dy_0\right)\right] - k\left(\frac{y(\omega,\tau^K)}{\omega}\right)\Big)\Big) h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega \\
&+ \int_{\underline{\omega}^2}^{\omega_0} \Phi\left(v\left(y(\omega) - T(y(\omega)) + L^K - t(\omega,\boldsymbol{\theta}_2,L^K) - k\left(\frac{y(\omega)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega \\
&+ \int_{\omega_0}^{\omega_0+d\omega_0} \Phi\left(v\left(y(\omega) - T(y(\omega)) + L^K - \tau(y(\omega) - y_0) - t(\omega,\boldsymbol{\theta}_2,L^K) - k\left(\frac{y(\omega)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega \\
&+ \int_{\omega_0+d\omega_0}^{\overline{\omega}^2} \Phi\left(v\left(y(\omega) - T(y(\omega)) + L^K - \tau dy_0 - t(\omega,\boldsymbol{\theta}_2,L^K) - k\left(\frac{y(\omega)}{\omega}\right)\right)\right) h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega \\
&+\lambda\int_{\underline{\omega}^1}^{\overline{\omega}^1} \left(T(y(\omega,\tau^K)) + \tau dy_0\right) h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega + \lambda\int_{\underline{\omega}^2}^{\omega_0} T(y(\omega,\tau^K))\, h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega \\
&+\lambda\int_{\omega_0}^{\omega_0+d\omega_0} \left(T(y(\omega,\tau^K)) + \tau(y - y_0)\right) h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega + \lambda\int_{\omega_0+d\omega_0}^{\overline{\omega}^2} \left(T(y(\omega,\tau^K)) + \tau dy_0\right) h(\omega|\boldsymbol{\theta}_2)\,\mathrm{d}\omega.
\end{aligned}$$

With $L^K = \int_{\underline{\omega}^1}^{\overline{\omega}^1} t^K\Big(y(\omega,\tau^K) - T(y(\omega,\tau^K)) - \tau dy_0\Big) h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega.$

$\dfrac{d\mathcal{L}}{d\tau}$ will have the same form as above except that now the derivative of the second term is:

$$\int_{\omega'^K}^{\overline{\omega}^1} \left(dy_0\tau^K(1-\gamma^K) - dy_0 - \frac{dt}{d\tau}\right) \Phi'(v)\, u'\, h(\omega|\boldsymbol{\theta}_1)\,\mathrm{d}\omega$$

and $\dfrac{dL^K}{d\tau} = -dy_0(1-\gamma^K)\tau^K\Big(H(\overline{\omega}^1|\boldsymbol{\theta}_1) - H(\omega'^K|\boldsymbol{\theta}_1)\Big).$

So since $\gamma^K > 1$, signs are inverted: taxes decrease the gap between the highest skilled of the extracting group and the individuals to whom the kinship tax applies.

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